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# Fundamental Characteristics and Applicability of the RADAR Method: Proof of Ranking Consistency

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**ABSTRACT**

This paper presents a mathematical explanation of one of the Multi-Attribute Decision-Making (MADM) methods—the Ranking based on Distance and Range (RADAR) method—along with its modified variant, RADAR II. Through mathematical proofs, the influence of each step of the method on the final ranking of alternatives is analyzed. The methods are tested on three numerical examples with varying criterion weights. The robustness of the methods, as well as their fundamental characteristics, is demonstrated. A comparative analysis reveals that although both methods prioritize alternatives based on their stability across all criteria—particularly the most important ones—the RADAR II method is somewhat more rigorous and stringent, whereas the original RADAR method is more flexible and yields more objective results.

## 1. Introduction

Multi-Attribute Decision-Making (MADM) is a mathematical tool applied across various scientific domains. In the literature, it is also referred to as Multi-Attribute Decision Analysis (MADA). Generally, MADM encompasses a set of mathematical methods designed for solving discrete optimization problems. More broadly, along with Multi-Objective Decision Making (MODM), it falls under the category of Multi-Criteria Decision-Making (MCDM) methods and, as such, can be classified within the domain of Operational Research (OR).

The fundamental characteristic of MADM methods is that a predefined set of alternatives is evaluated based on a predefined set of attributes/criteria. Therefore, they solve discrete optimization problems.

A large number of MADM methods have been developed and are widely used in the relevant literature. Likewise, different authors have proposed various classifications of these methods. One of the earliest and most fundamental classifications of MADM methods was presented in the

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publication by [1]. The author categorizes all MADM methods into three main groups: (1) methods for determining relevant criteria and alternatives, (2) methods for establishing the importance (weights) of criteria, and (3) methods for ranking alternatives.

Methods for determining criterion weights are frequently used in the literature. Although some of them can also be used for ranking alternatives, their primary purpose is to assign weights to criteria. Some of the most well-known methods include Analytic Hierarchy Process (AHP) [2,3], Analytic Network Process (ANP) [4], CRiteria Importance Through Intercriteria Correlation (CRITIC) [5], Decision-Making Trial and Evaluation Laboratory (DEMATEL) (see [6,7]), and Best-Worst Method (BWM) [8], as well as some newer methods such as Level Based Weight Assessment (LBWA) [9], Stochastic Identification of Weights (SITW) [10], Defining Interrelationships between Ranked Criteria (DIBR) [11], DIBR II [12,13], and others.

The most commonly used method from this group is AHP. It has been applied for determining criterion weights in various types of problems, such as supplier selection [14], site selection [15], and production planning problems [16], among others.

The largest group of methods consists of those used for ranking alternatives. This group includes methods based on different mathematical principles. Some of the most well-known methods are Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) [17], Multi-Criteria Optimization and Compromise Solution (in Serbian: VIšekriterijumska Optimizacija i Kompromisno Rešenje – VIKOR) [18], Multi-Attributive Border Approximation area Comparison (MABAC) [19], Measurement Alternatives and Ranking according to the COmpromise Solution (MARCOS) [20], Evaluation based on Distance from Average Solution (EDAS) [21], Additive Ratio Assessment (ARAS) [22], (Multi-Attributive RealIdeal Comparative Analysis (MARICA) [23], RAnking based on Distance And Range (RADAR) [24,25], and others.

These methods have a broad and diverse range of applications. For instance, they have been utilized for supplier selection [14,20], in the field of Information Technology [26], the oil and gas industry [27], wastewater treatment technologies [28], improving manufacturing process reliability [29], risk assessment [30], material selection [31], and engineering management [12], among others.

The selection of an appropriate MADM method depends on numerous factors related to the nature of the problem itself. It cannot be stated that one method is always superior to another; rather, certain methods are more suitable for specific types of problems.

The aim of this paper is to provide fundamental mathematical explanations of the RADAR method. The method was first introduced in [24], where it was applied to prioritize failure modes in the automotive industry. Subsequently, it was utilized for industrial equipment selection [25], where a new variant of the method (RADAR II) was also presented. Additionally, through the application of fuzzy set theory, the method has been extended and integrated into the Process Failure Mode and Effect Analysis (PFMEA) framework in [32].

The key difference between RADAR and RADAR II lies in the normalization process, specifically in determining the maximum and minimum proportion matrices, which may later impact the ranking of alternatives. The objective is to illustrate these differences through numerical examples. Both RADAR and RADAR II are well-suited for reliability-related problems, as they identify the most stable solution across all considered criteria. Furthermore, the mathematical foundation of the method is designed to mitigate the influence of alternatives that perform exceptionally well on less important criteria while effectively highlighting the quality of alternatives that perform well on critical criteria.

The fundamental concept and characteristics of the method are outlined in [24]. In this study, a mathematical proof is provided, and the operational principles of the method are explained. In addition, through numerical examples, the key features of RADAR and RADAR II are demonstrated.

Following the introductory chapter, Chapters 2 to 5 provide a mathematical explanation of the method's functioning. Chapter 6 presents numerical examples along with a discussion of the obtained results. Finally, Chapter 7 summarizes the key findings of this research.

## 2. Algorithm for applying the RADAR method

The fundamental steps of the RADAR method were first introduced in the study by [24], while the modification of the method and the RADAR II variant were first published in Komatina [25]. To ensure a clearer explanation of the method throughout the rest of the paper, this chapter presents the fundamental steps of both the RADAR and RADAR II methods.

Let us consider a set of alternatives $\{1, \ldots, i, \ldots I\}$, evaluated according to a set of criteria $\{1, \ldots, j, \ldots J\}$. The steps of the RADAR method can then be presented as follows [24,25]:

*Step 1.* Formation of the decision matrix:

$$\left[M_{ij}\right]_{I\times J} \tag{1}$$

*Step 2.* The maximum proportion matrix, $\alpha$:

$$\left[\alpha_{ij}\right]_{I\times J} \tag{2}$$

For the benefit type of criteria (RADAR):

$$\alpha_{ij} = \frac{\frac{\max_i M_{ij}}{M_{ij}}}{\left(\left(\frac{\max_i M_{ij}}{M_{ij}}\right)+\left(\frac{M_{ij}}{\min_i M_{ij}}\right)\right)} \tag{3}$$

For the cost type of criteria (RADAR):

$$\alpha_{ij} = \frac{\frac{M_{ij}}{\min_i M_{ij}}}{\left(\left(\frac{\max_i M_{ij}}{M_{ij}}\right)+\left(\frac{M_{ij}}{\min_i M_{ij}}\right)\right)} \tag{4}$$

For the benefit type of criteria (RADAR II):

$$\alpha_{ij} = \frac{\max_i M_{ij}-M_{ij}}{\left(\left(\max_i M_{ij}-M_{ij}\right)+\left(M_{ij}-\min_i M_{ij}\right)\right)} \tag{5}$$

For the cost type of criteria (RADAR II)

$$\alpha_{ij} = \frac{M_{ij}-\min_i M_{ij}}{\left(\left(\max_i M_{ij}-M_{ij}\right)+\left(M_{ij}-\min_i M_{ij}\right)\right)} \tag{6}$$

*Step 3.* The minimum proportion matrix, $\beta$:

$$\left[\beta_{ij}\right]_{I\times J} \tag{7}$$

For the benefit type of criteria (RADAR):

$$\beta_{ij} = \frac{\frac{M_{ij}}{\min_i M_{ij}}}{\left(\left(\frac{\max_i M_{ij}}{M_{ij}}\right)+\left(\frac{M_{ij}}{\min_i M_{ij}}\right)\right)} \tag{8}$$

For the cost type of criteria (RADAR):

$$\beta_{ij} = \frac{\frac{\max_i M_{ij}}{M_{ij}}}{\left(\left(\frac{\max_i M_{ij}}{M_{ij}}\right)+\left(\frac{M_{ij}}{\min_i M_{ij}}\right)\right)} \quad (9)$$

For the benefit type of criteria (RADAR II):

$$\beta_{ij} = \frac{M_{ij}-\min_i M_{ij}}{\left(\left(\max_i M_{ij}-M_{ij}\right)+\left(M_{ij}-\min_i M_{ij}\right)\right)} \quad (10)$$

For the cost type of criteria (RADAR II)

$$\beta_{ij} = \frac{\max_i M_{ij}-M_{ij}}{\left(\left(\max_i M_{ij}-M_{ij}\right)+\left(M_{ij}-\min_i M_{ij}\right)\right)} \quad (11)$$

From the given information, the following rules can be concluded:

- $a_{ij}$ for a benefit-type criteria is calculated in the same way as $\beta_{ij}$ for a cost-type criteria. The reverse also holds.
- For every considered $i, i = 1, \dots I$ evaluated according to any $j = 1, \dots, J$, the following holds: $a_{ij} + \beta_{ij} = 1$.

*Step 4.* The empty range matrix:

$$\left[E_{ij}\right]_{I\times J} \quad (12)$$

where

$$E_{ij} = \left|\alpha_{ij} - \beta_{ij}\right| \quad (13)$$

*Step 5.* The relative relationship matrix:

$$\left[RR_{ij}\right]_{I\times J} \quad (14)$$

where:

$$RR_{ij} = \frac{\alpha_{ij}}{\beta_{ij}+E_{ij}} \quad (15)$$

*Step 6.* The weighted relative relationship matrix:

$$\left[WRR_{ij}\right]_{I\times J} \quad (16)$$

Where:

$$WRR_{ij} = RR_{ij} \cdot \omega_j \quad (17)$$

*Step 7.* The aggregated ranking index, $RI_i$:

$$RI_i = \frac{\min \sum_{j=1}^{J} WRR_i}{\sum_{j=1}^{J} WRR_i} \quad (18)$$

The values of $RI_i$ need to be sorted in a non-increasing order. The best alternative is the one with the highest $RI_i$ value, which is always 1. The lowest-ranked alternative is the one with the smallest value of this coefficient.

## 3. Normalization of values: α and β matrices

### *3.1 Range of α and β values*

The normalization procedure in the RADAR method is defined in a dual manner. Instead of a single value, two normalized values are obtained: the distance from the best solution, $\alpha$, and the distance from the worst solution, $\beta$.

In the basic RADAR method, the elements of the maximum proportion matrix $\left[a_{ij}\right]_{I\times J}$ are determined according to Eq. (3) and Eq. (4).

The value $a_{ij}$ is within the interval [0, 1]. A value closer to 0 indicates that alternative $i, i = 1, \dots, I$ concerning criterion$j, j = 1, \dots, J$ is closer to the best available value. A value of 1 indicates the opposite.

Let us consider that:

$$X = \frac{\max_i M_{ij}}{M_{ij}} \quad (19)$$

$$Y = \frac{M_{ij}}{\min_i M_{ij}} \quad (20)$$

It follows that the values of $a_{ij}$ are:

For the benefit type of criteria:

$$a_{ij} = \frac{X}{X+Y} \quad (21)$$

For the cost type of criteria:

$$a_{ij} = \frac{Y}{X+Y} \quad (22)$$

*Theorem 1.* Monotonicity of $a_{ij}$ with respect to $X$ for a benefit-type criterion.

*Proof:* In the case where $X$ increases while the value of $Y$ remains constant, the value of $a_{ij}$ increases. Let us consider the function in Eq. (21).

In that case, the first derivative of the function $a_{ij}$ with respect to $X$ according to the quotient rule, can be written as follows:

$$\frac{\partial a_{ij}}{\partial X} = \frac{\partial}{\partial X} \quad \frac{X}{X+Y} = \frac{(X+Y)\cdot\frac{d}{dX}(X) - X\cdot\frac{d}{dX}(X+Y)}{(X+Y)^2} \quad (23)$$

Since $Y$ is constant, the following holds:

$$\frac{d}{dX}(X) = 1$$

$$\frac{d}{dX}(X+Y) = 1$$

From this, it follows that:

$$\frac{\partial a_{ij}}{\partial X} = \frac{(X+Y)\cdot 1 - X\cdot 1}{(X+Y)^2} = \frac{Y}{(X+Y)^2} \quad (24)$$

As $X > 0$ and $Y > 0$ always hold, it follows that:

$$(X+Y)^2 > 0$$

Therefore:

$$\frac{\partial a_{ij}}{\partial X} > 0$$

In this way, it is proven that $a_{ij}$ increases as the value of $X$ increases. In other words, the greater the difference between $\max_i M_{ij}$ for a given criterion and the considered alternative $M_{ij}$, the higher the value of $a_{ij}$, meaning that the alternative is further from the best solution.

*Theorem 2.* Monotonicity of $a_{ij}$ with respect to $Y$ for benefit-type criteria.

*Proof:* In the case where $Y$ increases while the value of $X$ remains constant, the value of $a_{ij}$ decreases.

Let us consider the function in Eq. (21). In this case, the first derivative of the function $a_{ij}$ with respect to $Y$ can be expressed using the quotient rule as follows:

$$\frac{\partial a_{ij}}{\partial Y} = \frac{\partial}{\partial Y} \quad \frac{X}{X+Y} = \frac{(X+Y)\cdot\frac{d}{dY}(X) - X\cdot\frac{d}{dY}(X+Y)}{(X+Y)^2} \quad (25)$$

Since $X$ is constant, the following holds:

$$\frac{d}{dY}(X) = 0$$

$$\frac{d}{dY}(X+Y) = 1$$

From this, it follows that:

$$\frac{\partial a_{ij}}{\partial Y} = \frac{(X+Y)\cdot 0 - X\cdot 1}{(X+Y)^2} = \frac{-X}{(X+Y)^2} \quad (26)$$

Since $X > 0$ and $Y > 0$ always hold, it follows that:

$$(X+Y)^2 > 0$$

Therefore:

$$\frac{\partial a_{ij}}{\partial Y} = \frac{-X}{(X+Y)^2} < 0$$

In this way, it is proven that $a_{ij}$ decreases as the value of $Y$ increases. In other words, the greater the difference between $M_{ij}$ and the worst alternative according to a given criterion, $\min_i M_{ij}$, the lower the value of $a_{ij}$, meaning that the alternative is closer to the best solution.

Both of these theorems also hold for cost-type criteria. However, $a_{ij}$ for cost-type criteria is calculated in the same way as $\beta_{ij}$ for benefit-type criteria. The reverse also applies.

*Example.* These theorems can also be tested on a simple numerical example. Let us consider an initial value of $X = 10$ and $Y = 5$. The criterion in question is of the benefit type. In this case:

$$\alpha = \frac{10}{10+5} = 0.667$$

$$\beta = \frac{5}{10+5} = 0.333$$

If we consider that the value of $Y$ remains constant while $X$ increases to 15, we obtain:

$$\alpha = \frac{15}{15+5} = 0.75$$

$$\beta = \frac{5}{15+5} = 0.25$$

In this way, Theorem 1 is also proven through an example, where an increase in the value of $X$ leads to an increase in $\alpha$, provided that $Y$ remains constant. In other words, the alternative is worse, i.e., it is further from the best alternative. The opposite also holds: if the value of $X$ decreases, $\alpha$ also decreases, assuming a constant $Y$.

If $X$ remains constant (initially 10) and $Y$ increases to 20, then we obtain:

$$\alpha = \frac{10}{10+20} = 0.333$$

$$\beta = \frac{20}{10+20} = 0.667$$

In this way, Theorem 2 is also proven by example. When the value of $Y$ increases while $X$ remains constant, the value of $\alpha$ decreases. The opposite also holds.

*3.2 Interdependence of α and β values*

*Theorem 3.* Mathematical proof that $\alpha + \beta = 1$.

*Proof:* The sum $\alpha + \beta$ can be expressed as follows:

$$\alpha + \beta = \frac{X}{X+Y} + \frac{Y}{X+Y} \quad (27)$$

Since both $\alpha$ and $\beta$ have the same denominators, we can simply add the numerators:

$$\alpha + \beta = \frac{X+Y}{X+Y} \quad (28)$$

From the given expression, it follows that:

$$\alpha + \beta = 1$$

In all situations from *Example*, the stated rule holds. Thus, it can be said that this theorem has also been tested on a numerical example.

The difference between the basic RADAR method and the RADAR II method lies only in the way $\alpha$ and $\beta$ are determined. Instead of using a ratio, the difference between the maximum and the considered value, as well as the difference between the considered value and the minimum, is taken, respectively. Although the mathematical operation has changed, the dependency of the variables remains the same and does not affect their relationship.

Therefore, in the RADAR II method, $\alpha$ and $\beta$ are determined according to Eq. (5) and Eq. (6), and also Eq. (10) and Eq. (11), respectively.

The fundamental difference between these two approaches is that the RADAR II method allows $\alpha$ and $\beta$ to take a value of 1. In the basic RADAR method, this is not possible because they are always:

$X + Y > X \quad \vee \quad X + Y > Y$

In this case, the numerator and the denominator are never equal, so the values of $\alpha$ and $\beta$ cannot be 1. Likewise, the numerator cannot be 0 when the denominator is not 0.

In the RADAR II method, the following rules apply (for the benefit-type criterion):

- If $M_{ij} = \max_i M_{ij}$, then $\alpha_{ij} = 0$. This occurs when the considered value is simultaneously the maximum value.
- If $M_{ij} = \min_i M_{ij}$, then $\alpha_{ij} = 1$. This occurs when the considered value is simultaneously the minimum value.

For $\beta_{ij}$ the opposite holds. Additionally, $\alpha$ and $\beta$ are inverse for the cost-type criteria.

From the above, it follows that for both the RADAR and RADAR II methods:

$\alpha = 1 - \beta \quad \wedge \quad \beta = 1 - \alpha$

The above holds for both benefit-type and cost-type criteria.

When comparing the approaches used in the RADAR and RADAR II methods, the following conclusions can be drawn:

- The RADAR method allows for finer adjustment of the values of $\alpha$ and $\beta$, which can later influence the final ranking of an alternative. In the RADAR method, these values also depend on the range of values within the considered criterion.
- RADAR II more clearly highlights the advantages and disadvantages of an alternative concerning a given criterion, respectively. This means that this approach is less flexible and tends to favour better alternatives while giving weaker alternatives fewer chances to achieve a higher overall ranking.

Both approaches have certain advantages and disadvantages. However, the choice of method largely depends on the type of problem being considered, as explained in this paper.

## 4. The empty range and relative relationship matrix

Through steps 4 and 5 of the RADAR (and RADAR II) method application algorithm, the Empty Range and Relative Relationship Matrix are calculated. These two matrices are interconnected, which is why they are explained together.

The values of the Empty range matrix, $[E_{ij}]_{I\times J}$, are calculated using the Eq. (13).

*Theorem 4.* Mathematical proof that the values of $E_{ij}$ are within the interval [0, 1].

*Proof:* Since it has been proven that the values of $\alpha_{ij}$ and $\beta_{ij}$ lie within the interval [0, 1], their sum is always 1, and their difference $\alpha_{ij} - \beta_{ij}$ is always within the interval [-1, 1]. The absolute value of this difference is therefore within the interval [0, 1]. Consequently, it follows that $E_{ij} \in [0, 1]$.

The values of the Relative Relationship Matrix, $[RR_{ij}]_{I\times J}$, are calculated using the Eq. (15).

*Theorem 5.* Mathematical proof that the values of $RR_{ij}$ are in the interval [0, 1].

*Proof:* It has been proven that the values of $\alpha_{ij}$ and $\beta_{ij}$ are within the interval [0, 1], that $\alpha_{ij} + \beta_{ij} = 1$, and that $E_{ij} = |\alpha_{ij} - \beta_{ij}| \in [0, 1]$. To prove that the values of $RR_{ij}$ are within the interval [0, 1] two cases need to be examined: $\alpha_{ij} \geq \beta_{ij}$ and $\beta_{ij} > \alpha_{ij}$.

*First case, $\alpha_{ij} \geq \beta_{ij}$:*

In this case, it is known that:

$$\alpha_{ij} \geq 0.5$$

Thus:

$$E_{ij} = |\alpha_{ij} - \beta_{ij}| = \alpha_{ij} - \beta_{ij}$$

When these values are applied in the formula for $RR_{ij}$ (Eq. (15)) the following conclusion can be derived:

$$RR_{ij} = \frac{\alpha_{ij}}{\beta_{ij} + E_{ij}} = \frac{\alpha_{ij}}{\beta_{ij} + \alpha_{ij} - \beta_{ij}} = \frac{\alpha_{ij}}{\alpha_{ij}} = 1$$

This proves that for $\alpha_{ij} \geq 0.5$, the value of $RR_{ij}$ is always equal to 1.

*Second case*, $\beta_{ij} > \alpha_{ij}$:

In this case, it is known that:

$$\alpha_{ij} < 0.5$$

Thus:

$$E_{ij} = |\alpha_{ij} - \beta_{ij}| = \beta_{ij} - \alpha_{ij}$$

When these values are applied in the formula for $RR_{ij}$ the following conclusion can be derived:

$$RR_{ij} = \frac{\alpha_{ij}}{\beta_{ij} + E_{ij}} = \frac{\alpha_{ij}}{\beta_{ij} + \beta_{ij} - \alpha_{ij}} = \frac{\alpha_{ij}}{2\beta_{ij} - \alpha_{ij}}$$

Since the following rule holds:

$$\beta_{ij} = 1 - \alpha_{ij}$$

We obtain the following expression:

$$\frac{\alpha_{ij}}{2\beta_{ij} - \alpha_{ij}} = \frac{\alpha_{ij}}{2 \cdot (1 - \alpha_{ij}) - \alpha_{ij}} = \frac{\alpha_{ij}}{2 - 3\alpha_{ij}}$$

Thus, the final result is:

$$RR_{ij} = \frac{\alpha_{ij}}{2 - 3\alpha_{ij}}$$

Since in this case $\alpha_{ij} \in [0, 0.5)$, the given expression for $RR_{ij}$ yields a positive value less than 1. For boundary values, the following holds:

- If $\alpha_{ij} = 0$, then $RR_{ij} = 0$;
- If $\alpha_{ij} = 0.5$, then $RR_{ij} = 1$;

This proof can also be illustrated with a numerical example shown in Table 1.

**Table 1**
Numerical Example of the Impact of $\alpha_{ij}$ and $\beta_{ij}$ on the value of $RR_{ij}$

| $\alpha_{ij}$ | $\beta_{ij}$ | $E_{ij}$ | $RR_{ij}$ |
|---|---|---|---|
| 0 | 1 | 1 | 0 |
| 0.1 | 0.9 | 0.8 | 0.06 |
| 0.2 | 0.8 | 0.6 | 0.14 |
| 0.3 | 0.7 | 0.4 | 0.27 |
| 0.4 | 0.6 | 0.2 | 0.5 |
| 0.5 | 0.5 | 0 | 1 |
| 0.6 | 0.4 | 0.2 | 1 |
| 0.7 | 0.3 | 0.4 | 1 |

**Table 1**
Continued

| $\alpha_{ij}$ | $\beta_{ij}$ | $E_{ij}$ | $RR_{ij}$ |
|---|---|---|---|
| 0.8 | 0.2 | 0.6 | 1 |
| 0.9 | 0.1 | 0.8 | 1 |
| 1 | 0 | 1 | 1 |

In this way, it has been proven that the values of $RR_{ij}$ always lie within the interval [0, 1].

Analyzing the values of $RR_{ij}$ leads to the conclusion that any alternative closer to the minimum value, i.e., with $\beta_{ij} \geq 0.5$, will have $RR_{ij} = 1$. This characteristic of the RADAR method supports the "stability" of the solution.

Thus, the final ranking is significantly influenced by alternatives that have $\alpha_{ij} > 0.5$ for a larger number of criteria. In other words, these are the alternatives that are "above average" for multiple criteria. However, the impact of criterion weights can also be significant, as explained in the following section.

## 5. Ranking of alternatives

According to step 6 of the proposed algorithm in the RADAR method, as well as in the RADAR II variant, weighting of values is performed. This step is carried out only when the considered criteria have different levels of importance.

Therefore, in this step, the values of the weighted relative relationship matrix, $[WRR_{ij}]_{I\times J}$, are determined using Eq. (17).

The final ranking of alternatives is determined based on the aggregated ranking index, $RI_i$, which is calculated using Eq. (18).

The highest value of $RI_i$ is always equal to 1. This value is assigned to the best alternative (there may be more than one). The ranking of alternatives is obtained by sorting them in a non-increasing order. The last alternative in the ranking is the one with the lowest $RI_i$ value.

*Theorem 6.* Mathematical proof of ranking consistency through the weighting of $RR_{ij}$ values.

*Proof:* Since $WRR_{ij}$ is calculated using the expression $WRR_{ij} = RR_{ij} \cdot \omega_j$, where the values of $RR_{ij} \in [0, 1]$, and the criteria ,weights $\omega_j \in (0, 1]$, it follows that the range of $WRR_{ij}$ is:

$0 \leq WRR_{ij} \leq \omega_j$

Where the rule holds that:

$$\sum_{j=1}^{J} \omega_j = 1$$

Let $\sum_{j=1}^{J} WRR_i$ be denoted as $S_i$ for clearer interpretation, then according to the expression for $RI_i$ we obtain:

$$RI_i = \frac{\min S_i}{S_i} \quad (29)$$

Then it holds that:

$S_i \geq \min S_i$

From this, it follows that:

$$RI_i = \frac{\min S_i}{S_i} \leq 1$$

Where $RI_i = 1$ at the value $S_i = \min S_i$ and is assigned to the best alternative.

Since $\omega_j > 0$ and $RR_{ij} \geq 0$ always hold, it follows that $S_i > 0$ for every considered alternative, $i, i = 1 \ldots, I$.

Let us test the impact of $\omega_j$ on the final ranking of alternatives. If there exist two alternatives, $a$ and $b$, for which it is known that $S_b > S_a$, where neither of them is $\min S_i$, then:

$$\frac{S_{min}}{S_a} > \frac{S_{min}}{S_b}$$

From this, it follows that:

$$RI_a > RI_b$$

Which means that alternative $a$ is ranked higher than alternative $b$.

The presented theorem can be illustrated with a numerical example through the application of the RADAR method. If we have the following decision matrix:

$$\begin{bmatrix} & \omega_1 = 0.4 & \omega_2 = 0.35 & \omega_3 = 0.25 \\ & j=1 & j=2 & j=3 \\ i=1 & 1 & 1 & 1 \\ i=2 & 1 & 1 & 1 \\ i=3 & 1 & 1 & 1 \end{bmatrix} \Rightarrow \begin{matrix} RI_1 = 1 \\ RI_2 = 1 \\ RI_3 = 1 \end{matrix}$$

However, if each alternative is the best according to one of the criteria (having a value of 2), the values of $RI_i$ change accordingly:

$$\begin{bmatrix} & \omega_1 = 0.4 & \omega_2 = 0.35 & \omega_3 = 0.25 \\ & j=1 & j=2 & j=3 \\ i=1 & 2 & 1 & 1 \\ i=2 & 1 & 2 & 1 \\ i=3 & 1 & 1 & 2 \end{bmatrix} \Rightarrow \begin{matrix} RI_1 = 1 \\ RI_2 = 0.96 \\ RI_3 = 0.88 \end{matrix}$$

From the given example, it can be observed that although the alternatives have equal values of $RI_1 = 1$ when criterion weights are not considered, applying the weighting process makes the best alternative the one that performs best according to the most important criterion, $j = 1$. The second-best alternative is the one that excels in the second most important criterion, $j = 2$, while the third-place alternative is the one that performs best in the least important criterion, $j = 3$.

In this way, even with a simple numerical example, it is demonstrated that criterion weights influence the ranking of alternatives. The better an alternative performs in a more important criterion, the higher the likelihood of achieving a better position in the final ranking.

## 6. Numerical examples and comparison of RADAR and RADAR II

After explaining the fundamental characteristics of the RADAR method and its variant, RADAR II, this chapter provides numerical examples to illustratively present and explain some of the features of these two methods.

### *6.1 Numerical example 1*

Let the supplier selection problem be considered hypothetically. A total of five potential suppliers, $\{1, \ldots, i, \ldots I\}$ are evaluated based on four relevant criteria, $\{1, \ldots, j, \ldots J\}$. The criteria are of different natures: unit procurement cost $(j = 1)$ in euros, delivery time $(j = 2)$ in days, product quality $(j = 3)$ on a scale from 1 to 10, and supplier flexibility $(j = 4)$ on a scale from 1 to 10. The first two criteria are cost-type, while the latter two are benefit-type. The data on suppliers are provided in Table 2.

**Table 2**
Decision matrix

| | $j=1$ | $j=2$ | $j=3$ | $j=4$ |
|---|---|---|---|---|
| $i=1$ | 12.5 | 2.4 | 9 | 4 |
| $i=2$ | 14.9 | 2.6 | 10 | 6 |
| $i=3$ | 11.3 | 1.5 | 9 | 9 |
| $i=4$ | 9.6 | 3.0 | 6 | 7 |
| $i=5$ | 11.5 | 2.0 | 8 | 10 |

Let us consider that the criteria weights are known: $\omega_1 = 0.35$, $\omega_2 = 0.2$, $\omega_3 = 0.3$, and $\omega_4 = 0.15$.

First, the elements of the maximum proportion matrix, $\alpha$ and the minimum proportion matrix, $\beta$ were calculated for the application of the RADAR and RADAR II methods. These values are given in Tables 3 and 4, respectively.

**Table 3**
The maximum and minimum proportion matrix – RADAR

| | $\alpha$ | | | | $\beta$ | | | |
|---|---|---|---|---|---|---|---|---|
| | $j=1$ | $j=2$ | $j=3$ | $j=4$ | $j=1$ | $j=2$ | $j=3$ | $j=4$ |
| $i=1$ | 0.522 | 0.561 | 0.426 | 0.714 | 0.478 | 0.439 | 0.574 | 0.286 |
| $i=2$ | 0.608 | 0.600 | 0.375 | 0.526 | 0.392 | 0.400 | 0.625 | 0.474 |
| $i=3$ | 0.472 | 0.333 | 0.426 | 0.331 | 0.528 | 0.667 | 0.574 | 0.669 |
| $i=4$ | 0.392 | 0.667 | 0.625 | 0.449 | 0.608 | 0.333 | 0.375 | 0.551 |
| $i=5$ | 0.480 | 0.471 | 0.484 | 0.286 | 0.520 | 0.529 | 0.516 | 0.714 |

Example of calculating the first element of the $\alpha$ and $\beta$ matrix (RADAR):

$$a_{11} = \frac{\frac{12.5}{9.6}}{\frac{14.9}{12.5} + \frac{12.5}{9.6}} = \frac{1.302}{1.192 + 1.302} = 0.522$$

$$\beta_{11} = \frac{\frac{14.9}{12.5}}{\frac{14.9}{12.5} + \frac{12.5}{9.6}} = 0.478$$

**Table 4**
The maximum and minimum proportion matrix – RADAR II

| | $\alpha$ | | | | $\beta$ | | | |
|---|---|---|---|---|---|---|---|---|
| | $j=1$ | $j=2$ | $j=3$ | $j=4$ | $j=1$ | $j=2$ | $j=3$ | $j=4$ |
| $i=1$ | 0.547 | 0.600 | 0.250 | 1.000 | 0.453 | 0.400 | 0.750 | 0.000 |
| $i=2$ | 1.000 | 0.733 | 0.000 | 0.667 | 0.000 | 0.267 | 1.000 | 0.333 |
| $i=3$ | 0.321 | 0.000 | 0.250 | 0.167 | 0.679 | 1.000 | 0.750 | 0.833 |
| $i=4$ | 0.000 | 1.000 | 1.000 | 0.500 | 1.000 | 0.000 | 0.000 | 0.500 |
| $i=5$ | 0.358 | 0.333 | 0.500 | 0.000 | 0.642 | 0.667 | 0.500 | 1.000 |

Example of calculating the first element of the $\alpha$ and $\beta$ matrix (RADAR II):

$$a_{11} = \frac{12.5 - 9.6}{(14.9 - 12.5) + (12.5 - 9.6)} = \frac{2.9}{2.4 + 2.9} = 0.547$$

$$\beta_{11} = \frac{14.9 - 12.5}{(14.9 - 12.5) + (12.5 - 9.6)} = 0.453$$

Table 5 presents the weighted relative relationship matrix obtained using the RADAR method, while Table 6 presents the weighted relative relationship matrix obtained using the RADAR II method.

**Table 5**
The weighted relative relationship matrix – RADAR

| | $j=1$ | $j=2$ | $j=3$ | $j=4$ |
|---|---|---|---|---|
| $i=1$ | 0.350 | 0.200 | 0.176 | 0.150 |
| $i=2$ | 0.350 | 0.200 | 0.129 | 0.150 |
| $i=3$ | 0.282 | 0.067 | 0.176 | 0.049 |
| $i=4$ | 0.166 | 0.200 | 0.300 | 0.103 |
| $i=5$ | 0.301 | 0.160 | 0.265 | 0.038 |

Example of calculating the first element of the weighted relative relationship matrix (RADAR):

$$E_{11} = |\alpha_{11} - \beta_{11}| = 0.522 - 0.478 = 0.044$$

$$RR_{11} = \frac{\alpha_{11}}{\beta_{11} + E_{11}} = \frac{0.522}{0.478 + 0.044} = 1$$

$$WRR_{11} = RR_{11} \cdot \omega_1 = 1 \cdot 0.35 = 0.35$$

**Table 6**
The weighted relative relationship matrix – RADAR II

| | $j=1$ | $j=2$ | $j=3$ | $j=4$ |
|---|---|---|---|---|
| $i=1$ | 0.350 | 0.200 | 0.060 | 0.150 |
| $i=2$ | 0.350 | 0.200 | 0.000 | 0.150 |
| $i=3$ | 0.108 | 0.000 | 0.060 | 0.017 |
| $i=4$ | 0.000 | 0.200 | 0.300 | 0.150 |
| $i=5$ | 0.136 | 0.067 | 0.300 | 0.000 |

Example of calculating the first element of the weighted relative relationship matrix (RADAR II):

$$E_{11} = |\alpha_{11} - \beta_{11}| = 0.547 - 0.453 = 0.094$$

$$RR_{11} = \frac{\alpha_{11}}{\beta_{11} + E_{11}} = \frac{0.547}{0.453 + 0.094} = 1$$

$$WRR_{11} = RR_{11} \cdot \omega_1 = 1 \cdot 0.35 = 0.35$$

The aggregated ranking index, $RI_i$, as well as the ranking of the considered alternatives using the RADAR and RADAR II methods, is given in Table 7.

**Table 7**
Ranking of alternatives (Numerical example 1)

| | *RADAR* | | *RADAR II* | |
|---|---|---|---|---|
| | $RI_i$ | *Rank* | $RI_i$ | *Rank* |
| $i=1$ | 0.655 | 5 | 0.243 | 5 |
| $i=2$ | 0.693 | 4 | 0.264 | 4 |
| $i=3$ | 1.000 | 1 | 1.000 | 1 |
| $i=4$ | 0.746 | 3 | 0.284 | 3 |
| $i=5$ | 0.753 | 2 | 0.368 | 2 |

Example of calculating the $RI_1$ parameter for RADAR:

$$RI_1 = \frac{\min(0.876, 0.829, 0.574, 0.770, 0.763)}{0.876} = \frac{0.574}{0.876} = 0.655$$

Example of calculating the $RI_1$ parameter for RADAR II:

$$RI_1 = \frac{\min(0.760, 0.700, 0.185, 0.650, 0.502)}{0.760} = \frac{0.185}{0.760} = 0.243$$

In the considered case, the alternatives take the same ranking position using both the RADAR and RADAR II methods.

### *6.2 Numerical example 2*

Let the same example be considered. In this case, the criteria weights have been changed, and their values are: $\omega_1 = 0.2$, $\omega_2 = 0.3$, $\omega_3 = 0.3$, and $\omega_4 = 0.2$. The aggregated ranking index, $RI_i$, as well as the ranking of the considered alternatives using the RADAR and RADAR II methods, is given in Table 8.

**Table 8**
Ranking of alternatives (Numerical example 2)

| | *RADAR* | | *RADAR II* | |
|---|---|---|---|---|
| | $RI_i$ | *Rank* | $RI_i$ | *Rank* |
| $i = 1$ | 0.574 | 5 | 0.190 | 4 |
| $i = 2$ | 0.607 | 3 | 0.206 | 3 |
| $i = 3$ | 1.000 | 1 | 1.000 | 1 |
| $i = 4$ | 0.604 | 4 | 0.180 | 5 |
| $i = 5$ | 0.693 | 2 | 0.302 | 2 |

In this case, the ranking of the alternatives remains the same, except that alternatives $i = 1$ and $i = 4$ have swapped places as the last and second-to-last in the ranking.

### *6.3 Numerical example 3*

In the third example, the following criteria weights were taken into account: $\omega_1 = 0.15$, $\omega_2 = 0.25$, $\omega_3 = 0.2$, and $\omega_4 = 0.4$. The aggregated ranking index, $RI_i$, as well as the ranking of the considered alternatives using the RADAR and RADAR II methods, is given in Table 9.

**Table 9**
Ranking of alternatives (Numerical example 3)

| | *RADAR* | | *RADAR II* | |
|---|---|---|---|---|
| | $RI_i$ | *Rank* | $RI_i$ | *Rank* |
| $i = 1$ | 0.494 | 5 | 0.156 | 4 |
| $i = 2$ | 0.512 | 4 | 0.164 | 3 |
| $i = 3$ | 1.000 | 1 | 1.000 | 1 |
| $i = 4$ | 0.568 | 3 | 0.154 | 5 |
| $i = 5$ | 0.748 | 2 | 0.383 | 2 |

In the third example, there are somewhat greater changes in the ranking of alternatives. However, even in this case, the top two alternatives remain unchanged in the ranking. The key change is observed for alternative $i = 4$, which is ranked third using the RADAR method, while it holds the fifth position when the RADAR II method is applied.

### *6.4 Discussion*

Based on the presented examples, the key conclusion is that there are no significant deviations in the ranking of alternatives when the criterion weights change. This indicates that the obtained solution is reliable and that the methods are robust. Figure 1 illustrates the changes in the ranking of alternatives across the three numerical examples when applying the RADAR method.

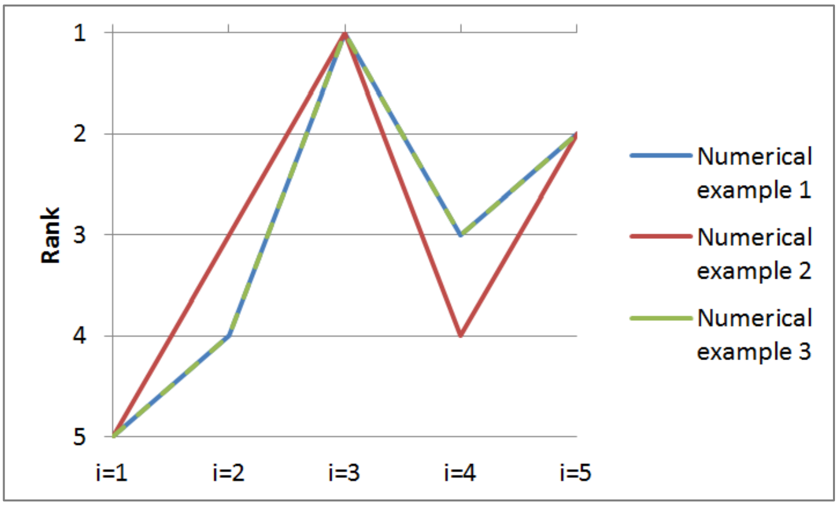


**Fig. 1.** Changes in the ranking of alternatives when applying the RADAR method

In the same way, Figure 2 illustrates the changes in the ranking of alternatives when using the RADAR II method.

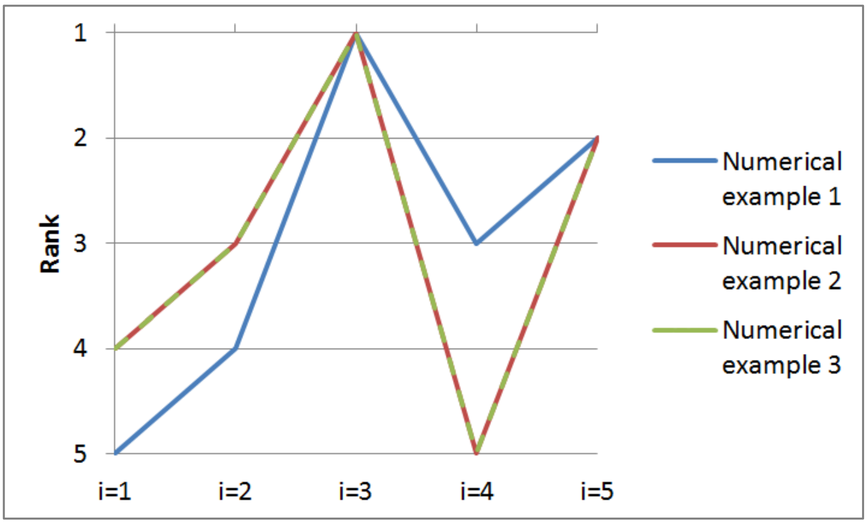


**Fig. 2.** Changes in the ranking of alternatives when applying the RADAR II method

In Figures 1 and 2, it is clearly observed that there is no significant difference in the changes in the ranking of alternatives. In fact, when applying the RADAR method, numerical examples 1 and 3 yield the same ranking. The same applies to the RADAR II method, but for numerical examples 2 and 3. It is important to emphasize that the top two alternatives in the ranking do not change places in any of the six considered cases.

Furthermore, the ranking comparison can also be conducted for each numerical example individually. Since the ranking in numerical example 1 is identical for both the RADAR and RADAR II methods, Figures 3 and 4 illustrate only the changes for numerical examples 2 and 3, respectively.

As seen in Figure 3, in numerical example 2, the ranking of alternatives differs only for the two lowest-ranked alternatives, which switch places in this case. These are alternatives $i = 1$ and $i = 4$. The RADAR method favors $i = 4$ as an alternative that is sufficiently stable according to two criteria, $j = 1$ and $j = 4$, while RADAR II is somewhat stricter and considers it stable only with respect to $j = 1$, where it is also the best. This phenomenon can be confirmed based on the $RR_{ij}$ values.

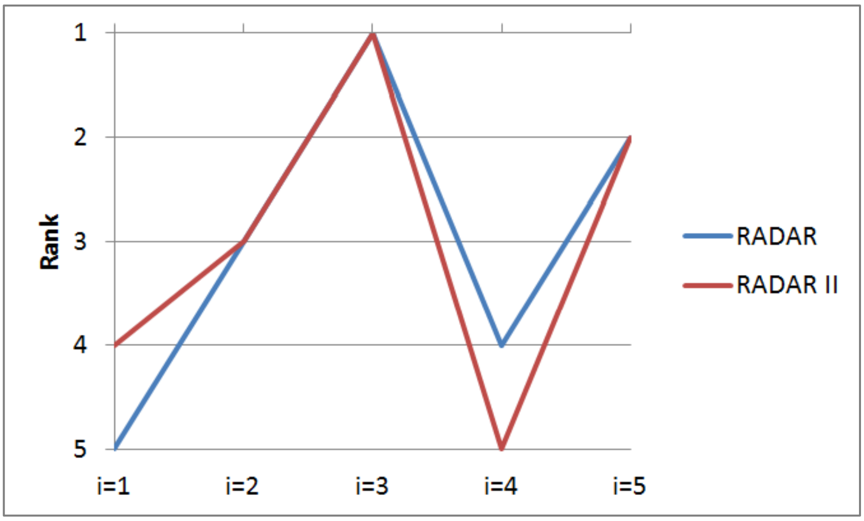


**Fig. 3.** Differences in the ranking of alternatives in numerical example 2

On the other hand, both methods perceive $i = 1$ as stable only concerning $j = 3$. However, since the weight of $j = 3$ is higher than that of $j = 1$, RADAR II ranks $i = 1$ better than $i = 4$. Therefore, the key difference lies in the lower tolerance of the RADAR II method for deviations of alternatives from the best solution.

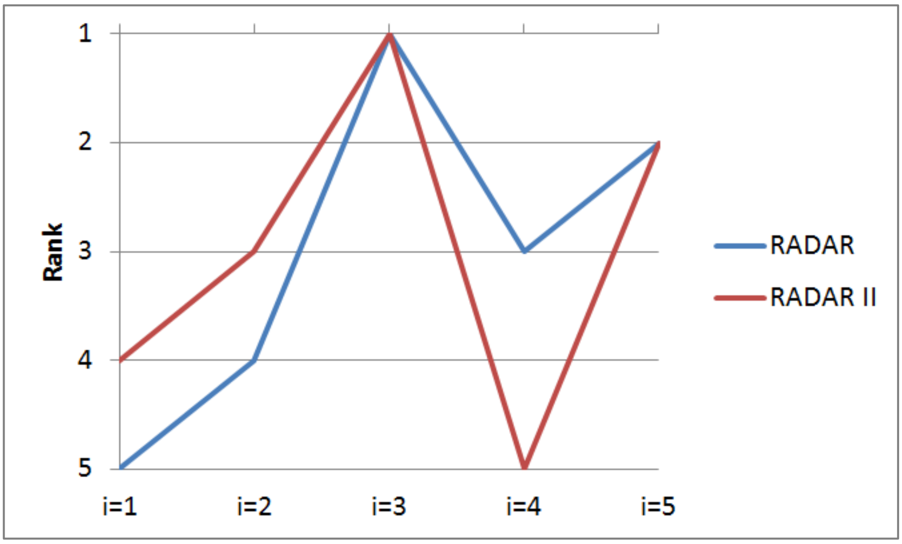


**Fig. 4.** Differences in the ranking of alternatives in numerical example 3

In the third numerical example (Figure 4), the greatest difference in the ranking of alternatives occurs. While the first and second-ranked alternatives do not change places, and the fourth and fifth alternatives maintain their relative positions, alternative i = 4 experiences a relatively large deviation. In the RADAR method, it is ranked third, whereas in RADAR II, it is placed last.

As established in the previous example, this alternative is stable according to criteria $j = 1$ and $j = 4$ in the RADAR method, whereas in RADAR II, it is stable only concerning $j = 1$. Since $j = 1$ is by far the least important criterion in this case, the RADAR II method does not recognize $i = 4$ as a relevant alternative. However, since the RADAR method identifies this alternative as stable also with respect to $j = 4$, which is the most important criterion in this case, it reaches third place in the ranking.

Thus, once again, the difference is due to the stricter nature of the RADAR II method, where it is significantly more difficult for alternatives to be considered acceptable according to the evaluated criteria.

From these examples, it can be concluded that the RADAR method is preferable when ranking precision is more important, allowing for more subtle differentiation among alternatives based on each criterion. However, if it is more crucial for the selected solution to be sufficiently stable according to as many important criteria as possible, the RADAR II method is the better choice.

## 7. Conclusions

In this study, a mathematical explanation of the RADAR and RADAR II methods is provided. The nature of these methods is examined through mathematical proofs of the given assumptions. In other words, the principles governing the methods' functionality are explained.

Since the key difference between the RADAR and RADAR II approaches lies in the determination of the maximum and minimum proportion matrices, the impact of this step on the final ranking of alternatives has been analyzed. The results indicate that the RADAR method is more suitable for slightly more precise ranking, allowing for somewhat greater deviations of alternative values from the best value for the considered criterion. On the other hand, RADAR II is significantly more rigorous and demands a higher level of "perfection." Nevertheless, both methods seek the most stable solution across all considered criteria.

Future research should focus on expanding the RADAR method by applying various techniques for determining criterion weights, as well as integrating different fuzzy approaches. Additionally, efforts should be made to apply the method in other economic sectors beyond the industrial domain.

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## Conflicts of Interest

The authors declare no conflicts of interest.

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