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Complex Fuzzy MARCOS and WASPAS Approaches with Z-Numbers for Augmented Reality Decision Making

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ABSTRACT

This study introduces two innovative decision-making methods, MARCOS and WASPAS, based on complex fuzzy $Z-$number sets. The methodology employs these approaches to evaluate the application of augmented reality (AR) in contemporary society as a case study, highlighting their advantages and nuanced differences. The complex fuzzy $Z-$number, which integrates $Z-$number and complex fuzzy set theories, serves as the foundation of this research. The study presents comprehensive flowcharts for both MARCOS and WASPAS, detailing their decision-making processes. A thorough analysis of the results is provided, along with future research directions that emphasize the potential of this methodological framework. The findings contribute to advancing decision-making in AR applications by offering comparative insights using complex fuzzy $Z-$number sets. Furthermore, the comparison section demonstrates the methodological robustness and validity of the proposed approach.

1. Introduction

Augmented Reality (AR) serves as a transformative force, introducing an enhanced, interactive dimension to the real-world environment through digital visual elements, sounds, and holographic technology. It emerges as a transformative force, extending beyond entertainment with products like Magic Leap One and global phenomena like Pokémon Go. Its applications span diverse sectors, addressing challenges in education, urban navigation, remote collaboration, and workplace training. AR also plays a pivotal role in decision-making, enhancing contextual awareness through innovative frameworks tailored for complex scenarios like Complex fuzzy Z-numbers. The evolving landscape of AR showcases its multifaceted impact on human experiences and decision processes. While the

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case study emphasizes how Augmented Reality (AR) enhances contextual awareness and decision-making through innovative frameworks, it also underscores practical applications for smartphones. AR applications on smartphones leverage technologies like GPS, providing real-time information on building addresses, retail offers, and restaurant reviews through electronic displays. This dual perspective highlights the broad spectrum of AR's impact, from refining decision processes in complex scenarios to facilitating everyday interactions through portable devices. Both AR and Multiple Criteria Decision Making (MCDM) [**?**] showcase a shared theme of utilizing advanced technologies to address challenges and optimize experiences, whether in complex decision-making contexts or in enhancing real-world information accessibility through handheld devices.

Examining the integration of Augmented Reality (AR) into Multiple Criteria Decision Making (MCDM), the study focuses on the nuanced complexities of Complex fuzzy Z-numbers (CFZNs). Karsak, describe MCDM as the process of categorizing alternatives based on criteria for optimal selection [1]. To address uncertainties Ashraf and other researchers, propose fuzzy MCDM using fuzzy numbers in decision-making [2–5]. The imprecision in rating alternatives during practical decision-making is linked to the instinctive nature of human thought [6]. Rodríguez et al. [7] highlight the development of diverse mechanisms and theories to manage uncertainty in practical problems. MCDM is pivotal in selecting optimal alternatives, with decision makers (DMs) using various evaluation processes, such as crisp, interval, and fuzzy numbers [8]. Sometime authors used group decision making instead of decision making analysis in large scale [9, 10]. Case studies used for decision making local consensus adjustment [11], trust risk relation [12].

Making decisions (DM) is a complex process that includes selecting options based on a variety of standards and requirements. Numerous research papers on DM have been written in the past. Examples include risk calculations in oil and gas supply [13], diseases finding [14], material selection for nitrogen tanks [16], admission of students in department [17], and many more. The purpose of this paper is to extend the fuzzy set theory by presenting a new concept known as the Z-Numbers by using WASPAS [15] and MARCOS.

1.1 *Literature Review*

The foundational work of Zadeh [18] in 1965 marked the transition from classical sets to fuzzy sets, accompanied by the definition of key fuzzy set operators. Fuzzy numbers, endorsed by Chou et al. [19] and Fan [20], offer a more fitting representation for uncertain attribute values in MADM problems. Addressing unpredictability and uncertainty, Atanassov [21] introduced the concept of intuitionistic fuzzy sets (IFS), offering a successful methodology for effectively managing these factors. After this intuitionistic fuzzy set is used for decision making in three way [22]. This innovative concept provides a successful methodology for proficiently handling and navigating these intricate factors. Ramot [23] introduced a complex fuzzy set as a solution to address and navigate through these constraints. However, the aforementioned Fuzzy Set (FS) models fall short of adequately capturing periodic information or two-dimensional phenomena. To address this limitation, Ramot et al. [24] introduced the concept of Complex Fuzzy Sets (CFS). Unlike a traditional fuzzy membership function constrained to $[0, 1]$, the Complex Fuzzy Set extends its range to the unit circle in the complex plane, drawing researchers' attention and prompting the development of multiple theories associated with this unique concept [25]. The significance of the phase term is crucial in defining the Complex Fuzzy model [26], distinguishing it from other fuzzy set models. The uniqueness of CF sets lies in determining the membership value of an element through both an amplitude term and a phase term. Zadeh's introduction of the Z-number theory in [27] addresses limitations by integrating two sets of fuzzy numbers, enabling improved evaluation of events and enhanced reliability measurement for assessed values. The widespread application of Z-numbers, as evidenced in [28] [29], underscores their efficacy in handling uncertainties across diverse decision-making problems. Utilizing Z-numbers in decision-making enhances the reliability and significance of decision information. This study also delves into the closely linked aspect of Z-number ranking [30]. Subsequently, an evaluative analysis is performed to assess the efficacy of Z-numbers [31] in decision-making contexts. It effectively conveys the idea that Z-numbers are crucial in multiple criteria decision-making, and their significance is supported by substantial impact, as elaborated in key works [32] and [33]. After this, the hukuhara difference $Z-$number is introduced for the purpose of the difference of two $Z-$number [34].

Similarly, there are some methods and aggregation operators that are applied to decision-making in fuzzy set theory, like TODIM [35]. Zavadskas proposed the WASPAS approach, seamlessly integrating the advantages of both methods [36]. The WSM method contributes to the ease of alternative evaluation through the weighted sum, while the WMP method's advantage lies in preventing the acquisition of solutions with low values. Moreover, formulating the theory of the WASPAS technique poses a particularly intricate and challenging task for researchers in the field of fuzzy logic. This paper utilizes the Measurement of Alternatives and Ranking According to Compromise Solution (MARCOS) method to ascertain the most suitable solution in our case study [37]. The WASPAS method [38] and the MARCOS method [39] represent two classical approaches adept at addressing various Multiple Attribute Decision Making (MADM) problems effectively.

Motivation:
The motivation behind this research stems from the recognition of the critical role that decision-making plays in the successful deployment and utilization of AR technology. Traditional decision-making models often fall short in capturing the intricacies and uncertainties inherent in AR contexts, leading to suboptimal outcomes and diminished user experiences. By harnessing the power of Complex Fuzzy MARCOS and WASPAS approaches, augmented with Z-Numbers, this study endeavors to bridge this gap by offering a comprehensive framework capable of handling the multifaceted nature of AR decision scenarios.

Importance:
The significance of this research lies in its potential to significantly enhance the efficacy, reliability, and user satisfaction of AR applications across diverse domains. By equipping decision-makers with advanced tools capable of accommodating uncertainties, preferences, and priorities, this study contributes towards unlocking the full potential of AR technology in real-world settings. Moreover, the incorporation of Z-Numbers enriches the decision-making process by enabling a more nuanced representation of uncertainty, thereby facilitating more accurate and robust decision outcomes.

Future Direction:
The findings and methodologies presented in this research paper hold practical implications for various stakeholders involved in the development, deployment, and utilization of AR systems. From designers and developers seeking to optimize user experiences to policymakers and industry leaders striving to leverage AR technology for strategic decision-making, the insights gleaned from this study offer actionable guidance and frameworks for informed decision-making in AR contexts.

Here are some gaps in the past research that this paper fills:

- Handling Uncertainty in AR Decision Making: Previous research in AR decision-making often struggled to adequately address the inherent uncertainties present in real-world scenarios. This paper fills this gap by introducing the utilization of Z-Numbers within the Complex Fuzzy MARCOS and WASPAS approaches, providing a more robust framework for decision-making under uncertainty.

- Integration of Multi-Criteria Decision Making Techniques in AR: While AR technology has seen significant advancements, the integration of advanced decision-making techniques, such as Multi-Criteria Decision Making (MCDM), has been relatively limited. This study bridges this gap by demonstrating the applicability and effectiveness of Complex Fuzzy MARCOS and WASPAS approaches in the context of AR decision-making.

- Addressing Complexity in AR Applications: AR environments often exhibit complex and dynamic attributes, posing challenges for traditional decision-making models. By incorporating Complex Fuzzy MARCOS and WASPAS methodologies, this research fills the gap by providing a systematic approach to handle the complexity inherent in AR decision scenarios.

- Enhancing User Experience in AR Systems: Prior research has identified the importance of user experience in the adoption and success of AR applications. However, there remains a gap in terms of integrating advanced decision-making techniques to optimize user experiences. This paper addresses this gap by proposing a comprehensive framework that prioritizes user preferences and satisfaction within AR decision-making processes.

- Empirical Validation of Decision-Making Models in AR: While theoretical frameworks for AR decision-making exist, empirical validation in real-world scenarios is often lacking. This study fills this gap by providing empirical evidence of the efficacy and applicability of the proposed Complex Fuzzy MARCOS and WASPAS approaches with Z-Numbers through case studies and experimentation.

In this paper, we utilize a distance measure, investigating its application after a comprehensive study of distance metrics on complex sets, specifically for implementing on these two techniques [40]. Introducing a novel method, we apply these two approaches to our data, subsequently comparing the feasibility and validity of our outcome.

The presentation of our analysis takes the form of: Section 2 encompasses all the existing methodologies. Section 3 initiates by presenting the features of complex fuzzy Z-numbers, delving into the examination of scoring and ranking within the CFZNs framework. In Section 4, we elaborate on the derivation of the WASPAS method for CFZNs. Similarly, in Section 5, we explore the derivation of MARCOS techniques. In Section 6, the paper emphasizes the effectiveness of the Multiple Attribute Decision Making (MADM) strategy within the devised structures. In the subsequent subsections, we define an illustrative case for WASPAS techniques, followed by the definition of the representative case of MARCOS. Section 7 presents the study's conclusion.

2. Basic Terminologies

This section introduces fundamental terminology associated with Fuzzy Sets (IFSs) and Complex Fuzzy Sets (CFSs) across the universal set Γ. [18] Consider $FS\ H$ within the universal set Γ is determined through:

$$H = \{(\check{D}, \Re(\check{D}) | \Re \in \Gamma)\} \tag{1}$$

Here, the function is a mapping from Γ to the interval $[0, 1]$, and for any $\check{D} \in \Gamma, 0 \leq \Re(\check{D}) \leq 1$, the function $\Re(\check{D})$ is considered the membership function of within the set Γ. [23] Consider (CFS) on the set Γ is defined as:

$$= \{\check{D}, (\sigma_{\Re}(\check{D}) \exp\{2\pi i \tau_{\Re}(\check{D})\} : \check{D} \in \Gamma)\} \quad (2)$$

where $\check{D} : \Gamma \to \emptyset$ the membership function is symbolized as $\sigma_{\Re}\left(\check{D}\right) \exp\left\{2\pi i \tau_{\Re}\left(\check{D}\right)\right\} \forall \check{D} \in \Gamma$. Within the complex plane $\emptyset = \{r : r \in Ø_1, |r| \leq 1\}$ and $i = \sqrt{-1}, 0 \leq \left(\sigma_{\Re}\left(\check{D}\right), \tau_{\Re}\left(\check{D}\right)\right) \leq 1$. Considering two complex fuzzy sets, represented as $\ddot{Y}$= $\left\{\sigma_{\Re}\left(\check{D}\right) . \exp\left\{2\pi i\left(\tau_{\Re}\left(\check{D}\right)\right)\right\}\right\}$ and $\tilde{U}$=$\left\{\rho_t\left(\check{D}\right) . \exp\left\{2\pi i\left(\breve{o}_t\left(\check{D}\right)\right)\right\}\right\}$, the subsequent operations are outlined below:

1. $\ddot{Y} \cup \tilde{U} = \max\left(\sigma_{\Re}\left(\check{D}\right), \rho_t\left(\check{D}\right)\right) \exp\left\{\max 2\pi i\left(\tau_{\Re}\left(\check{D}\right), \breve{o}_{\{t}\left(\check{D}\right)\right)\right\};$
2. $\ddot{Y} \cap \tilde{U} = \min\left(\sigma_{\Re}\left(\check{D}\right), \rho_t\left(\check{D}\right)\right) \exp\left\{\min 2\pi i\left(\tau_{\Re}\left(\check{D}\right), \breve{o}_t\left(\check{D}\right)\right)\right\};$
3. $\ddot{Y}^c = 1 - \sigma_{\Re}\left(\check{D}\right) \exp\left\{2\pi i\left(1 - \left(\tau_{\Re}\left(\check{D}\right)\right)\right)\right\}:$

[29] An Z-number, indicated as Ř=(Q, Z), denotes a pair of fuzzy numbers. The primary component Q, represents a real-valued uncertain variable with a spectrum of values, while the secondary component, Z quantifies the reliability associated with the initial component. [40] A set of distance measures designed to quantify the dissimilarity between CFZNs $R = \{\sigma_{\acute{e}} \exp\{2\pi i(\tau_{\grave{e}})\}, \varpi_{\grave{e}} \exp\{2\pi i(\Re_{\grave{e}})\}\}$ and $V = \{\sigma_{\grave{\imath}} \exp\{2\pi i(\tau_{\grave{\imath}})\}, \varpi_{\grave{\imath}} \exp\{2\pi i(\Re_{\grave{\imath}})\}\}$, provides a systematic way to assess the divergence between the two sets.

$$\Omega_{\Re} = (R_{\grave{e}}, V_{\grave{\imath}})$$
$$= \tfrac{1}{2}((\sigma_{\grave{e}}.\varpi_{\grave{e}}) - (\sigma_{\grave{\imath}}.\varpi_{\grave{\imath}}))^2 + \tfrac{1}{4}(\pi)^2((\tau_{\grave{e}}.\Re_{\grave{e}}) - (\tau_{\grave{\imath}}.\Re_{\grave{\imath}}))^2$$

3. CFZN Information

In this section, we have introduced the concept of CFZN Complex Fuzzy $Z-$Number along with its fundamental properties, scoring and ranking. Consider the universal set Γ, where represents the complex fuzzy $Z-$numbers, in accordance with the following condition described below:

$= \{\sigma_{\Re}(\check{D}) \exp\{2\pi i \tau_{\Re}(\check{D})\},$
$\varpi_{\Re}(\check{D}) \exp\{2\pi i \Re_{\Re}(\check{D})\}\}$ For instance, : $\Gamma arrow[0, 1]$ which represents the ordered pair of membership and reliability associated with its complex terms. The conditions stipulate that $(\sigma_{\Re}, \varpi_{\Re}) \in [0, 1]$ and $(\tau_{\Re}, \Re_{\Re}) \in [0, 1]$ must be satisfied. Let $_a = \{\sigma_1 \exp\{2\pi i(\tau_1)\}, \varpi_1 \exp\{2\pi i(\Re_1)\}\}$ and $_b = \{\sigma_2 \exp\{2\pi i(\tau_2)\}, \varpi_2 \exp\{2\pi i(\Re_2)\}\}$ are two CFZN and $\lambda > 0$. Then consecutive relations are presented:

1. $_1 \supseteq_2 \Leftrightarrow \sigma_1 \exp\{2\pi i(\tau_1)\} \geq \sigma_2 \exp\{2\pi i(\tau_2)\}, \varpi_1 \exp\{2\pi i(\Re_1)\} \geq \varpi_2 \exp\{2\pi i(\Re_2)\},$
2. $_1 =_2 \Leftrightarrow \sigma_2 \exp\{2\pi i(\tau_2)\} \supseteq \sigma_1 \exp\{2\pi i(\tau_1)\}$,$\sigma_1 \exp\{2\pi i(\tau_1)\} \supseteq \sigma_2\{\exp 2\pi i(\tau_2)\}$ and $\varpi_2 \exp\{2\pi i(\Re_2)\} \supseteq \varpi_1 \exp\{2\pi i(\Re_1)\}$,$\varpi_1 \exp\{2\pi i(\Re_1)\} \supseteq \varpi_2\{\exp 2\pi i(\Re_2)\},$
3. $_1 \cup_2 = \{\max(\sigma_1, \sigma_2)\} \exp\{\max 2\pi i(\tau_1), 2\pi i(\tau_2)\}$ and $\{\max(\varpi_1, \varpi_2)\} \exp\{\max 2\pi i(\Re_1), 2\pi i(\Re_2)\},$
4. $_1 \cap_2 = \{\min(\sigma_1, \sigma 2)\} \exp\{\min 2\pi i(\tau_1), 2\pi i(\tau_2)\}$ and $\{\min(\varpi_1, \varpi_2)\} \exp\{\min 2\pi i(\Re_1), 2\pi i(\Re_2)\},$
5. $^C_1 = \{(1 - \sigma_1) \exp\{2\pi i(1 - \tau_1)\}, (1 - \varpi_1) \exp\{2\pi i(1 - \Re_1)\}\};$
6. $_1 \oplus_2 = \{\sigma_1 + \sigma_2 - \sigma_1\sigma_2 \exp\{2\pi i(\tau_1 + \tau_2 + \tau_1\tau_2)\}, \varpi_1 + \varpi_2 - \varpi_1\varpi_2 \exp\{2\pi i(\Re_1 + \Re_2 + \Re_1\Re_2)\}\},$
7. $_1 \otimes_2 = \{\sigma_1\sigma_2 \exp\{2\pi i(\tau_1, \tau_2)\}, \varpi_1\varpi_2 \exp\{2\pi i(\Re_1, \Re_2)\}\},$
8. $\lambda_1 = \{1 - (1 - \sigma_1)^\lambda \exp\{2\pi i(1 - (1 - \tau_1)^\lambda)\}, 1 - (1 - \varpi_1)^\lambda \exp\{2\pi i(1 - (1 - \Re_1)^\lambda)\}\},$
9. $^\lambda_1 = \{\{\sigma_1 \exp\{2\pi i(\tau_1)\}\}^\lambda, \{\varpi_1 \exp\{2\pi i(\Re_1)\}\}^\lambda\},$

For each Complex Fuzzy Z-Number (CFZN),define the scoring function $\acute{R}_{\Re}$ and the accuracy function $\beta_{\Re}$ applicable to the element $\Re$,as explained below:

$$\mu(\acute{R}_{\Re}) = \frac{(\sigma_{\Re}.\varpi_{\Re}) + (\tau_{\Re}.\Re_{\Re})}{2} \tag{3}$$

$$\delta(\acute{L}_{\Re}) = \frac{(\sigma_{\Re}.\varpi_{\Re}) - (\tau_{\Re}.\Re_{\Re})}{2} \tag{4}$$

In the context of alternatives, ensure that $\mu(\acute{R}_{\Re}) \in [0,1]$ and $\delta(\acute{L}_{\Re}) \in [0,1]$. Additionally,the CFZN $\acute{R}_x$ is regarded as more superior than another $\acute{R}_y$ represented by $\acute{R}_x > \acute{R}_y$,if either $\mu(\acute{R}_x) > \mu(\acute{R}_y)$ or $\mu(\acute{R}_x = \mu(\acute{R}_y)$ and $\delta(\acute{L}_x) > \delta(\acute{L}_y)$ hold.

4. WASPAS Method

The WASPAS method which stands for Weighted Aggregated Sum Product Assessment, integrates two previously mentioned techniques: the Weighted Sum Model (WSM)the Weighted Product Model (WMP) [41]. This method, conceived by Zavadskas in 2012, was further extended to incorporate fuzzy logic in 2016. The WASPAS technique holds significance for its ability to effectively handle complex decision scenarios and provide robust solutions in multiple criteria decision-making (MCDM) processes [42], [43].

Algorithm

1. Expert decision matrix.
2. The technique utilizes input data in the format of a matrix that captures alternatives and criteria. This matrix is constructed using information gathered from expert input. In the provided decision matrix, where represents the number of alternatives and $\Re$ represents the number of criteria, the element $T_{\Re}$ denotes the performance of the $\Re th$ alternative in relation to the th criterion.
3. Normalized value of decision matrix in equations below. The formula representing the Benefit criteria can be articulated as:
$$T_{\Re}^{!`} = \frac{T_{\Re}}{\max T_{\Re}} \tag{5}$$
The formula representing the Cost criteria can be articulated as:
$$T_{\Re}^{!`} = \frac{T_{\Re}}{\min T_{\Re}} \tag{6}$$
4. Deriving the weighted normalized decision fuzzy matrix for WSM Equation (7) and WPM Equation (8) involves performing specific calculations.
$$WSM = T_{\Re} = T_{\Re}^{!`} W_{\Re} \tag{7}$$
$$WPM = T_{\Re} = (T_{\Re}^{!`})^{W_{\Re}} \tag{8}$$

5. Determining the optimality function values for WSM Equation (9) and WPM Equation (10) involves computing their respective optimization metrics.

$$\breve{E}^{WSM} = \sum_{\Re=1}^{m} T_{\Re}^{!`} W_{\Re} \tag{9}$$

$$\breve{E}^{WPM} = \prod_{\Re=1}^{m} (T_{\Re}^{!`})^{W_{\Re}} \tag{10}$$

6. The overall significance of the ith alternative is assessed by calculating the Weighted Sum (WS) and Weighted Product (WP) through equations (9) and (10), respectively.

$$\breve{E}^{WASPAS} = \Upsilon \breve{E}^{WSM} + (1 - \Upsilon) \breve{E}^{WPM} \tag{11}$$

Here, $\breve{E}^{WSM}$ and $\breve{E}^{WPM}$ represent the relative significance of the th alternative with respect to the $\Re$ th criterion, determined using the WS and WP methods, respectively. The weight ($W_{\Re}$) assigned to each criterion is considered and is the parameter $\Upsilon = 1$. The weights assigned by individual experts to each criterion are then averaged. After that, we acquire the score values in this step.

7. Rank the alternatives and identify the most favorable option by referencing the score value $\breve{E}^{WASPAS}$ in above Equation (11).

Figure 1 illustrates the visual representation of the accomplished work. Additional details on the procedures are expounded within the framework of Figure 1.

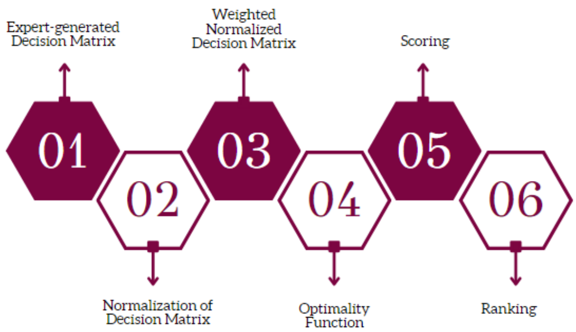

Fig. 1. Flowchart of WASPAS method

5. MARCOS Method

In the following section, we present a concise overview of the MARCOS method. The MARCOS method introduces a fresh perspective to multicriteria analysis, centered around a unique relationship between alternatives and their reference values. These reference values represent both ideal and negative points. Decision-making within the MARCOS method [44] relies on a utility function, which serves as an alternative measure in comparison to the positive and negative ideal solutions [45]. The method identifies the best alternative as the one that is closest to the ideal point while simultaneously being farthest from the anti-ideal reference point.

Algorithm

1. The first step involves creating the initial decision matrix for $\Re$ alternatives based on criteria.

2. Expanding the initial fuzzy decision matrix involves augmenting the matrix with both negative-ideal (Δ) and ideal solution (Λ). The negative-ideal solution (Δ) corresponds to an alternative characterized by the least desirable attributes, while the ideal solution (Λ) represents an alternative with the most favorable characteristics. The determination of the negative-ideal solution (Δ) is achieved through the application of the following expression:

$$\Delta = \left\{ \begin{array}{c} \min \sigma_{\Re} \exp\{\min 2\pi i(\tau_{\Re})\}, \\ \min \varpi_{\Re} \exp\{\min 2\pi i(\Re_{\Re})\} \\ if \quad \in B \\ \max \sigma_{\Re} \exp\{\max 2\pi i(\tau_{\Re})\}, \\ \max \varpi_{\Re} \exp\{\max 2\pi i(Re_{\Re})\} \\ if \ \in C \end{array} \right\}.$$

The acquisition of the ideal solution (Λ) is accomplished through the utilization of the following expression:

$$\Lambda = \left\{ \begin{array}{c} \max \sigma_{\Re} \exp\{\max 2\pi i(\tau_{\Re})\}, \\ \max \varpi_{\Re} \exp\{\max 2\pi i(\Re_{\Re})\} \\ if \ \in B \\ \min \sigma_{\Re} \exp\{\min 2\pi i(\tau_{\Re})\}, \\ \min \varpi_{\Re} \exp\{\min 2\pi i(\Re_{\Re})\} \\ if \ \in C \end{array} \right\}.$$

B denotes the criteria categorized as benefits, aiming for maximization, whereas C signifies the criteria categorized as costs, targeting minimization.

3. Regarding distance measurement, the following formulas are provided by drawing motivation for the determination of both ideal and negative distance measures.

$$\Omega_{\Re}^{+} = (R_{\grave{e}}, V_{\grave{\imath}})$$
$$= \Big\{ \Big(\frac{1}{2} \Big(\big((\sigma_{\grave{e}}.\varpi_{\grave{e}}) - (\sigma_{\grave{\imath}}.\varpi_{\grave{\imath}}) \big)^2 + \frac{1}{4(\pi)^2} \big((\tau_{\grave{e}}.\Re_{\grave{e}}) - (\tau_{\grave{\imath}}.\Re_{\grave{\imath}}) \big)^2 \Big) \Big) \Big\} \tag{12}$$

$$\Omega_{\Re}^{-} = (R_{\grave{e}}, V_{\grave{\imath}})$$
$$= \Big\{ \Big(\frac{1}{2} \Big(\big((\sigma_{\grave{e}}.\varpi_{\grave{e}}) - (\sigma_{\grave{\imath}}.\varpi_{\grave{\imath}}) \big)^2 + \frac{1}{4(\pi)^2} \big((\tau_{\grave{e}}.\Re_{\grave{e}}) - (\tau_{\grave{\imath}}.\Re_{\grave{\imath}}) \big)^2 \Big) \Big) \Big\} \tag{13}$$

4. The closeness coefficient is established by employing $\Omega_{\Re}^{+}$ and $\Omega_{\Re}^{-}$, specified as follows:

$$\Phi_{\Re} = \frac{\Omega_{\Re}^{-}}{\Omega_{\Re}^{-} + \Omega_{\Re}^{+}} \tag{14}$$

5. Create the extended decision matrix by incorporating $\Phi_{\Re}$, along with the negative-ideal solution Ķ$^{-} = \{\Phi_{\bar{\Re}1}, \Phi_{\bar{\Re}2}, ..., \Phi_{\bar{\Re}n}\}$ and ideal solution Ķ$^{+} = \{\Phi^{+}_{\Re}; = 1, 2, ..., n\}$.

$$A = \begin{pmatrix} \Phi_{\bar{\Re}1} & \Phi_{\bar{\Re}2} & \dots & \Phi_{\bar{\Re}n} \\ \Phi_{11} & \Phi_{12} & \dots & \Phi_{1n} \\ \Phi_{21} & \Phi_{22} & \dots & \Phi_{2n} \\ \vdots & \vdots & \dots & \vdots \\ \Phi_{m1} & \Phi_{m2} & \dots & \Phi_{mn} \\ \Phi^{+}_{\Re1} & \Phi^{+}_{\Re2} & \dots & \Phi^{+}_{\Re n} \end{pmatrix} \tag{15}$$

Here

$$\Phi_{\bar{\Re}} = \left\{ \begin{array}{l} \min_{\Re} \Phi_{\Re},\ if\ \in B \\ \max_{\Re} \Phi_{\Re},\ if\ \in C \end{array} \right\}.$$

and

$$\Phi^{+}_{\Re} = \left\{ \begin{array}{l} \max_{\Re} \Phi_{\Re},\ if\ \in B \\ \min_{\Re} \Phi_{\Re},\ if\ \in C \end{array} \right\}.$$

Within the framework of the provided equations, labeled as Equation (5) and Equation (5), where in B and C signify the benefit and cost types respectively, the process involves selecting the maximum and minimum values in accordance with the definitions 3.

6. Normalize the extended decision matrix A into the form A´=$[\vartheta_{\Re}]_{(m+2)*n}$, using the following equations:

$$\vartheta_{\Re} = \frac{\Phi_{\Re}}{\Phi^{+}_{\Re}},\ if\ \in B \tag{16}$$

$$\vartheta_{\Re} = \frac{\Phi^{+}_{\Re}}{\Phi_{\Re}},\ if\ \in C \tag{17}$$

Formulate the normalization of the extended decision matrix A into A´, considering the elements $\Phi_{\Re}$ and $\Phi^{+}_{\Re}$ within the A matrix.

7. Formulate the final weighted decision matrix, denoted as Q=$[q_{\Re}]_{(m+2)*n}$, in accordance with Equation (18), where 'Z$_{\Re}$' stands as an element within the matrix A´, and 'w' denotes the weight associated with the '-th' criterion.

$$Q_{\Re} = Z_{\Re} * w \tag{18}$$

8. Ascertain the degree of utility for alternatives, denoted as $\Psi_{\Re}$, utilizing Equations (19) and (20).

$$\Psi^{-}_{\Re} = \frac{ß_{\Re}}{ß^{-}} \tag{19}$$

$$\Psi^{+}_{\Re} = \frac{ß_{\Re}}{ß^{+}} \tag{20}$$

Where ß$_{\Re} = \sum_{=1}^{n} Q_{(\Re+1)}$ $(\Re = 1, 2, ..., m)$ represents the utility degree, ß$^{+} = \sum_{=1}^{n} Q_{(m+2)}$ denotes the positive utility degree, and ß$^{-}$=$g \sum_{=1}^{n} Q_{1}$ signifies the negative utility degree.

9. Determine the utility function of alternatives, denoted as $\Gamma(\Psi_{\Re})$, through the computation based on the following equation.

$$\Gamma(\Psi_{\Re}) = \frac{\Psi_{\Re}^{+} + \Psi_{\Re}^{-}}{1 + \frac{1-\Gamma(\Psi_{\Re}^{+})}{\Gamma(\Psi_{\Re}^{+})} + \frac{1-\Gamma(\Psi_{\Re}^{-})}{\Gamma(\Psi_{\Re}^{-})}} \tag{21}$$

The utility function, denoted as $\Gamma(\Psi_{\Re})$, is determined in connection with the ideal state $\Gamma(\Psi_{\Re}^{+})$ and anti-ideal state $\Gamma(\Psi_{\Re}^{-})$, with their respective formulations presented as given in Equation (22) and Equation (23).

$$\Gamma(\Psi_{\Re}^{+}) = \frac{\Psi_{\Re}^{-}}{\Psi_{\Re}^{+} + \Psi_{\Re}^{-}} \tag{22}$$

$$\Gamma(\Psi_{\Re}^{-}) = \frac{\Psi_{\Re}^{+}}{\Psi_{\Re}^{+} + \Psi_{\Re}^{-}} \tag{23}$$

10. Arrange the alternatives in order of their utility function values, aiming for each alternative to possess the highest achievable utility function value.

Figure 2 exhibits the visual representation of the aforementioned methodology applied to CFZNs.

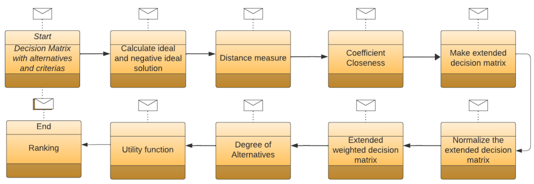

Fig. 2. Flowchart of MARCOS method

6. Case Study

In recent years, the transformative potential of Augmented Reality (AR) has emerged as a catalyst in reshaping various facets of human life [46]. The evolution of AR technology has transcended mere entertainment applications, extending its influence across domains that profoundly impact individuals' daily experiences. Mete Omerali apply COPRAS method by using spherical fuzzy set [47] in AR [48]. Noteworthy among these advancements is the development of AR headsets and glasses, exemplified by the Magic Leap One, a product that has garnered attention for its capacity to overlay digital content seamlessly onto the real world. Through strategic partnerships with entities like the NBA, Magic Leap has enabled users to engage in interactive and immersible experiences, allowing for the integration of digital data into the physical environment. The result is a paradigm shift, empowering users to witness life-size holographic representations, such as the virtual presence of iconic figures like Shaquille O'Neal, in their living spaces.
Beyond the realm of entertainment, the proliferation of AR apps and games has showcased the technology's potential to enrich human experiences. Pokémon Go, a globally embraced phenomenon, harnessed AR to merge the virtual and real worlds, captivating over a billion users. This success underscores AR's ability to transcend traditional boundaries and captivate a diverse global audience. Furthermore, AR has not only become an integral part of consumer engagement but has also found

application in content creation tools. Industry leaders like Adobe have developed sophisticated AR content creation tools, such as Adobe Aero, enabling companies like Adidas to deliver interactive and personalized experiences, as demonstrated through the creation of AR-driven sneaker design encounters for customers.

The significance of AR extends beyond the realm of entertainment and consumer engagement. It addresses a fundamental disconnect between the physical and digital worlds, enhancing human capabilities by providing a bridge between the two. The anticipated mainstream adoption of AR, with projected spending reaching $60 billion, heralds a new era wherein individuals can interact with information and insights derived from an increasingly interconnected and smart world. In sectors like healthcare, AR is already demonstrating its transformative potential, as seen in the case of AccuVein, where the technology converts the heat signature of a patient's veins into a visible image, significantly improving medical procedures' success rates.

As AR emerges as the new interface between humans and machines, it is poised to redefine how individuals learn, make decisions, and interact with their surroundings. The deployment of AR in various sectors, from manufacturing to healthcare, promises to amplify human capabilities, streamline operations, and create new possibilities for innovation. Pioneering organizations like Magic Leap, Adobe, and others are already navigating the deployment of AR to achieve tangible impacts on quality, productivity, and overall human experiences. As AR becomes an integral part of our daily lives, its role in transforming human existence is increasingly undeniable, offering a glimpse into a future where the boundaries between the physical and digital realms seamlessly converge for the betterment of society.

In conclusion, the ongoing journey of AR, spearheaded by innovators like Magic Leap and Apple, is fundamentally altering how individuals interact with and perceive the world. This transformative technology transcends entertainment, leaving an indelible mark on education, professional environments, and vital industries. As augmented reality becomes an integral interface connecting humans and technology, the seamless fusion of digital and physical realities anticipates a future where AR enhances human experiences, boosts productivity, and seamlessly integrates into the fabric of daily life. The case study illuminates the present landscape and envisions a future where augmented reality stands as a pivotal force shaping our interactions with the surrounding world.

Some of the attributes which are given by our experts

1. **Lack of Engagement in Education:** The lack of engagement in education is a pressing concern that hampers effective learning outcomes. Traditional teaching methods often fail to captivate students' attention, resulting in disinterest and reduced comprehension. This issue highlights the need for innovative approaches, such as incorporating technology like augmented reality (AR) into classrooms. AR can transform the educational experience by making lessons interactive and visually stimulating, encouraging active participation and fostering a deeper understanding of complex concepts. Embracing these advancements holds the potential to reinvigorate education, making it more engaging and impacting for students.

2. **Navigation Challenges in Urban Environments:** Navigation challenges in urban environments stem from the complexity of city layouts and the need for user-friendly guidance. Augmented reality (AR) navigation apps address these issues by overlaying digital information onto the real world, offering directional cues and highlighting points of interest. This innovation enhances urban navigation, providing users with an interactive and efficient way to navigate through bustling cityscapes.

3. **Limited Interaction in Remote Collaboration:** Limited interaction in remote collaboration hinders effective communication and teamwork. Augmented reality (AR) powered remote collaboration tools offer a solution by creating immersive virtual environments where team members

can interact with 3D models and engage in lifelike discussions. This advanced approach enhances the collaborative experience, making remote work more dynamic and engaging.

4. **Inefficient Training Processes in the Workplace:** TInefficient training processes in the workplace pose challenges in terms of time, cost, and effectiveness. Traditional training methods often lack practicality and real-world application, hindering employee skill development. Augmented reality (AR) emerges as a solution, offering immersive and realistic training simulations that streamline learning, reduce costs, and enhance overall workforce proficiency.

Alternatives:

1. **Augmented Reality Learning Platforms:** Augmented reality (AR) can transform the educational experience by overlaying digital content onto the real world. AR applications can bring textbooks to life, allowing students to interact with 3D models, conduct virtual experiments, and explore historical events in an immersive way. This fosters a more engaging and interactive learning environment, enhancing comprehension and retention.

2. **AR Navigation Apps:** AR navigation apps overlay digital information onto the real world, making it easier for users to navigate through cities. These apps can display directional arrows, highlight points of interest, and provide additional information about landmarks. By using the camera on a mobile device, users can see real-time information superimposed on their surroundings, improving navigation efficiency and enhancing the overall urban experience.

3. **AR-powered Remote Collaboration Tools:** AR facilitates more immersive remote collaboration by allowing individuals to interact with 3D models, share virtual whiteboards, and simulate face-to-face meetings. Virtual avatars and spatial audio enhance the feeling of presence, making remote collaboration more engaging and effective. This is particularly valuable for industries like design, where teams can collaboratively manipulate and discuss virtual prototypes.

4. **AR-based Training Simulations:** Augmented reality offers a solution by providing realistic training simulations. For example, in healthcare, AR can simulate surgeries, allowing medical professionals to practice procedures in a risk-free environment. Similarly, AR can be used in manufacturing for equipment operation training. This approach accelerates the learning process, reduces costs associated with physical training setups, and enhances the overall proficiency of workers.

Fig. 3. Augmented Reality Impact

Augmented reality (AR) is making significant strides in the realm of Multiple Criteria Decision Making (MCDM). By overlaying digital information onto the real-world environment, AR enhances decision-makers' contextual awareness and facilitates informed choices. Its immersive nature and ability to visualize complex data contribute to more effective and intuitive decision-making processes in various domains. The research presents an innovative decision-making framework tailored to address intricacies linked with Complex Fuzzy Z-Numbers (CFZNs). Examine a collection of data identified as $U_{\Re} = \{U_1, U_2, ..., U_m\}$ presented as a group of alternatives with corresponding attributes $Y = \{Y_1, Y_2, ..., Y_n\}$. To consistently facilitate the methodology, a crucial weight vector $W_{\Re}$ was required, where $\sum_{\Re=1}^{\zeta}$. In this exemplified scenario, we delineate features pertaining to benefit types. In the subsequent discussion, we will employ the proposed WASPAS and MARCOS approaches on Complex Fuzzy Z-Numbers (CFZNs) to address the Multiple Attribute Group Decision-Making (MAGDM) problem.

6.1 *Cases for Illustration*

Within this section, we embark on a renewed evaluation of the previously presented case study, illustrating the practicability and efficacy of our recommended WASPAS algorithms within the framework of Multiple Criteria Decision Making (MCDM).

Step 1: The method employs input information organized in the structure of a matrix, encompassing alternatives and criteria. This matrix is formed through insights provided by expert input.

Table 1
Decision Matrix Evaluated by Experts

	Y_1	Y_2
U_1	$\{0.3\exp\{2\pi i(0.6)\}, 0.3\exp\{2\pi i(0.6)\}\}$	$\{0.1\exp\{2\pi i(0.5)\}, 0.7\exp\{2\pi i(0.2)\}\}$
U_2	$\{0.2\exp\{2\pi i(0.3)\}, 0.2\exp\{2\pi i(0.4)\}\}$	$\{0.8\exp\{2\pi i(0.7)\}, 0.2\exp\{2\pi i(0.9)\}\}$
U_3	$\{0.6\exp\{2\pi i(0.8)\}, 0.1\exp\{2\pi i(0.1)\}\}$	$\{0.7\exp\{2\pi i(0.1)\}, 0.1\exp\{2\pi i(0.7)\}\}$
U_4	$\{0.2\exp\{2\pi i(0.7)\}, 0.5\exp\{2\pi i(0.2)\}\}$	$\{0.9\exp\{2\pi i(0.6)\}, 0.1\exp\{2\pi i(0.4)\}\}$
	Y_3	Y_4
U_1	$\{0.1\exp\{2\pi i(0.5)\}, 0.2\exp\{2\pi i(0.3)\}\}$	$\{0.2\exp\{2\pi i(0.5)\}, 0.4\exp\{2\pi i(0.2)\}\}$
U_2	$\{0.5\exp\{2\pi i(0.6)\}, 0.6\exp\{2\pi i(0.4)\}\}$	$\{0.3\exp\{2\pi i(0.6)\}, 0.4\exp\{2\pi i(0.2)\}\}$
U_3	$\{0.3\exp\{2\pi i(0.4)\}, 0.8\exp\{2\pi i(0.6)\}\}$	$\{0.1\exp\{2\pi i(0.2)\}, 0.1\exp\{2\pi i(0.7)\}\}$
U_4	$\{0.3\exp\{2\pi i(0.6)\}, 0.4\exp\{2\pi i(0.3)\}\}$	$\{0.1\exp\{2\pi i(0.2)\}, 0.4\exp\{2\pi i(0.9)\}\}$

Step 2: The equations above provide normalized values for the decision matrix concerning criteria of the benefit type.

Table 2
Normalized Decision Matrix

	Y_1	Y_2
U_1	$\{0.5\exp\{2\pi i(0.75)\}, 0.8\exp\{2\pi i(0.5)\}\}$	$\{0.1\exp\{2\pi i(0.8)\}, 1\exp\{2\pi i(0.2)\}\}$
U_2	$\{0.3\exp\{2\pi i(0.3)\}, 0.4\exp\{2\pi i(0.1)\}\}$	$\{0.8\exp\{2\pi i(1)\}, 0.2\exp\{2\pi i(1)\}\}$
U_3	$\{1\exp\{2\pi i(1)\}, 0.2\exp\{2\pi i(0.2)\}\}$	$\{0.7\exp\{2\pi i(0.1)\}, 0.1\exp\{2\pi i(0.7)\}\}$
U_4	$\{0.3\exp\{2\pi i(0.8)\}, 1\exp\{2\pi i(0.5)\}\}$	$\{1\exp\{2\pi i(0.8)\}, 0.1\exp\{2\pi i(0.7)\}\}$
	Y_3	Y_4
U_1	$\{0.2\exp\{2\pi i(0.8)\}, 0.2\exp\{2\pi i(0.5)\}\}$	$\{0.6\exp\{2\pi i(0.8)\}, 1\exp\{2\pi i(0.2)\}\}$
U_2	$\{1\exp\{2\pi i(1)\}, 0.7\exp\{2\pi i(0.6)\}\}$	$\{1\exp\{2\pi i(1)\}, 1\exp\{2\pi i(0.2)\}\}$
U_3	$\{0.6\exp\{2\pi i(0.6)\}, 1\exp\{2\pi i(1)\}\}$	$\{0.3\exp\{2\pi i(0.3)\}, 0.2\exp\{2\pi i(0.7)\}\}$
U_4	$\{0.6\exp\{2\pi i(1)\}, 0.5\exp\{2\pi i(0.5)\}\}$	$\{0.3\exp\{2\pi i(0.3)\}, 1\exp\{2\pi i(1)\}\}$

Step 3: Calculate the weighted normalized decision fuzzy matrix using WSM Equation (7) and WPM Equation (8) as defined in the algorithm.

Table 3
Weighted Normalized Decision Matrix for WSM by equation (6)

$U_{\Re}$	Y_1	Y_2
U_1	$\{0.15\exp\{2\pi i(0.3)\}, 0.08\exp\{2\pi i(0.1)\}\}$	$\{0.3\exp\{2\pi i(0.1)\}, 0.04\exp\{2\pi i(0.06)\}\}$
U_2	$\{0.1\exp\{2\pi i(0.15)\}, 0.04\exp\{2\pi i(0.2)\}\}$	$\{0.2\exp\{2\pi i(0.4)\}, 0.02\exp\{2\pi i(0.2)\}\}$
U_3	$\{0.3\exp\{2\pi i(0.4)\}, 0.02\exp\{2\pi i(0.05)\}\}$	$\{0.2\exp\{2\pi i(0.05)\}, 0.01\exp\{2\pi i(0.1)\}\}$
U_4	$\{0.1\exp\{2\pi i(0.35)\}, 0.1\exp\{2\pi i(0.1)\}\}$	$\{0.3\exp\{2\pi i(0.3)\}, 0.01\exp\{2\pi i(0.08)\}\}$
$U_{\Re}$	Y_3	Y_4
U_1	$\{0.06\exp\{2\pi i(0.3)\}, 0.02\exp\{2\pi i(0.1)\}\}$	$\{0.2\exp\{2\pi i(0.3)\}, 0.1\exp\{2\pi i(0.04)\}\}$
U_2	$\{0.3\exp\{2\pi i(0.4)\}, 0.07\exp\{2\pi i(0.13)\}\}$	$\{0.3\exp\{2\pi i(0.4)\}, 0.1\exp\{2\pi i(0.04)\}\}$
U_3	$\{0.1\exp\{2\pi i(0.2)\}, 0.1\exp\{2\pi i(0.2)\}\}$	$\{0.1\exp\{2\pi i(0.1)\}, 0.02\exp\{2\pi i(0.1)\}\}$
U_4	$\{0.1\exp\{2\pi i(0.4)\}, 0.05\exp\{2\pi i(0.1)\}\}$	$\{0.1\exp\{2\pi i(0.1)\}, 0.1\exp\{2\pi i(0.2)\}\}$

Table 4
Weighted Normalized Decision Matrix for WPM by equation (7)

$U_{\Re}$	Y_1	Y_2
U_1	$\{0.8\exp\{2\pi i(0.8)\}, 0.9\exp\{2\pi i(0.8)\}\}$	$\{0.5\exp\{2\pi i(0.9)\}, 1\exp\{2\pi i(0.7)\}\}$
U_2	$\{0.7\exp\{2\pi i(0.6)\}, 0.9\exp\{2\pi i(1)\}\}$	$\{0.9\exp\{2\pi i(1)\}, 0.8\exp\{2\pi i(1)\}\}$
U_3	$\{1\exp\{2\pi i(1)\}, 0.8\exp\{2\pi i(0.7)\}\}$	$\{0.9\exp\{2\pi i(0.4)\}, 0.8\exp\{2\pi i(0.9)\}\}$
U_4	$\{0.7\exp\{2\pi i(0.9)\}, 0.1\exp\{2\pi i(0.8)\}\}$	$\{1\exp\{2\pi i(0.9)\}, 0.8\exp\{2\pi i(0.8)\}\}$
$U_{\Re}$	Y_3	Y_4
U_1	$\{0.6\exp\{2\pi i(0.8)\}, 0.9\exp\{2\pi i(0.8)\}\}$	$\{0.8\exp\{2\pi i(0.9)\}, 1\exp\{2\pi i(0.7)\}\}$
U_2	$\{0.1\exp\{2\pi i(0.1)\}, 0.9\exp\{2\pi i(0.9)\}\}$	$\{1\exp\{2\pi i(1)\}, 1\exp\{2\pi i(0.7)\}\}$
U_3	$\{0.8\exp\{2\pi i(0.8)\}, 1\exp\{2\pi i(1)\}\}$	$\{0.7\exp\{2\pi i(0.6)\}, 0.8\exp\{2\pi i(0.9)\}\}$
U_4	$\{0.8\exp\{2\pi i(1)\}, 0.9\exp\{2\pi i(0.8)\}\}$	$\{0.7\exp\{2\pi i(0.6)\}, 1\exp\{2\pi i(1)\}\}$

Step 4: Determining the optimality function values for WSM and WPM by using equation (9) and(10) involves computing their respective optimization metrics.

Table 5
Optimality Function For WSM

$U_{\Re}$	Y_x
U_1	$\{0.44\exp\{2\pi i(1.30)\}, 0.30\exp\{2\pi i(0.28)\}\}$
U_2	$\{0.96\exp\{2\pi i(1.35)\}, 0.24\exp\{2\pi i(0.57)\}\}$
U_3	$\{0.81\exp\{2\pi i(0.85)\}, 0.15\exp\{2\pi i(0.56)\}\}$
U_4	$\{0.68\exp\{2\pi i(1.22)\}, 0.26\exp\{2\pi i(0.48)\}\}$

Table 6
Optimality Function For WPM

$U_{\Re}$	Y_x
U_1	$\{0.22\exp\{2\pi i(0.72)\}, 0.85\exp\{2\pi i(0.41)\}\}$
U_2	$\{0.69\exp\{2\pi i(0.67)\}, 0.78\exp\{2\pi i(0.68)\}\}$
U_3	$\{0.57\exp\{2\pi i(0.25)\}, 0.61\exp\{2\pi i(0.68)\}\}$
U_4	$\{0.44\exp\{2\pi i(0.57)\}, 0.07\exp\{2\pi i(0.64)\}\}$

Step 5: We obtain the score values in this phase by employing the Weighted Sum Method (WSM) and Weighted Product Method (WPM).

Table 7
Scoring

$U_{\Re}$	Y
U_1	0.25
U_2	0.50
U_3	0.30
U_4	0.38

Step 6: Evaluate the alternatives and determine the most favorable choice by considering the score value $\breve{E}^{WASPAS}$ obtained in the preceding Step 5.

$$(U_2) > (U_4) > (U_3) > (U_1)$$

6.2 *Representative Case:*

In this section, we undertake a thorough reassessment of the previously introduced case study, highlighting the practicality and effectiveness of our recommended MARCOS, algorithms within the framework of Multiple Criteria Decision Making (MCDM).

Step 1: The initial step entails generating the decision matrix for $\Re$ alternatives grounded on U criteria, and the resulting fuzzy decision matrix formation is illustrated in Table 1.

Step 2: Determine both the negative-ideal (Δ) and ideal solution (Λ) from the augmented matrix by utilizing Equations (2) and (2) as defined above.

$$\Delta = \left\{ \begin{array}{c} (0.6 \exp\{2\pi i(0.8)\}, 0.5 \exp\{2\pi i(0.4)\}), \\ (0.9 \exp\{2\pi i(0.7)\}, 0.7 \exp\{2\pi i(0.9)\}), \\ (0.5 \exp\{2\pi i(0.6)\}, 0.8 \exp\{2\pi i(0.6)\}), \\ (0.1 \exp\{2\pi i(0.2)\}, 0.4 \exp\{2\pi i(0.9)\}) \end{array} \right\}.$$

$$\Lambda = \left\{ \begin{array}{c} (0.2 \exp\{2\pi i(0.3)\}, 0.1 \exp\{2\pi i(0.1)\}), \\ (0.1 \exp\{2\pi i(0.1)\}, 0.1 \exp\{2\pi i(0.2)\}), \\ (0.1 \exp\{2\pi i(0.4)\}, 0.2 \exp\{2\pi i(0.3)\}), \\ (0.1 \exp\{2\pi i(0.2)\}, 0.1 \exp\{2\pi i(0.2)\}) \end{array} \right\}.$$

Step 3: Concerning distance measurement, the following equations are provided for the determination of both ideal and negative distance measures.

Table 8
Ideal distance by equation (12)

$U_{\Re}$	Y_1	Y_2	Y_3	Y_4
U_1	0.008	0.080	0.036	0.0006
U_2	0.017	0.055	0.002	0.002
U_3	0.015	0.080	0.006	0.0001
U_4	0.010	0.073	0.019	0.0001

Table 9
Negative Ideal distance by equation (13)

$U_{\Re}$	Y_1	Y_2	Y_3	Y_4
U_1	0.002	0.0009	0.000005	0.001
U_2	0.0001	0.007	0.0196	0.002
U_3	0.0004	0.0009	0.0121	0.001
U_4	0.001	0.001	0.002	0.0008

Step 4: Determine the closeness of coefficient.

Table 10
Closeness of coefficient by equation (14)

$U_{\Re}$	Y_1	Y_2	Y_3	Y_4
U_1	0.228	0.011	0.0001	0.715
U_2	0.008	0.126	0.883	0.568
U_3	0.026	0.011	0.652	0.883
U_4	0.138	0.022	0.112	0.846

Step 5: Generate the extended decision matrix.

Table 11
Extended decision matrix via equation (5) and (5)

$U_{\Re}$	Y_1	Y_2	Y_3	Y_4
U^-	0.008	0.011	0.0001	0.0001
U_1	0.228	0.011	0.0001	0.715
U_2	0.008	0.126	0.883	0.568
U_3	0.026	0.011	0.652	0.883
U_4	0.138	0.022	0.112	0.846
U^+	0.228	0.126	0.883	0.883

Step 6: In this step, the extended decision matrix is normalized.

Table 12
Extended Decision Matrix via equation (16)

$U_{\Re}$	Y_1	Y_2	Y_3	Y_4
U^-	0.037	0.089	0.0001	0.0001
U_1	1	0.094	0.0001	0.809
U_2	0.037	1	1	0.643
U_3	0.117	0.089	0.127	1
U_4	0.605	0.175	0.127	0.957
U^+	1	1	1	1

Step 7: Assess the weighted normalized extended decision matrix.

Table 13
Weighted normalized decision matrix via equation (18)

$U_{\Re}$	Y_1	Y_2	Y_3	Y_4
U^-	0.011	0.035	0.00001	0.00003
U_1	0.3	0.037	0.00001	0.1619
U_2	0.011	0.4	0.1	0.128
U_3	0.035	0.035	0.012	0.2
U_4	0.181	0.070	0.012	0.191
U^+	0.3	0.4	0.1	0.2

Step 8: Ascertain the degree of utility for alternatives, denoted as $\Psi_{\Re}$, utilizing Equations (19) and (20).

Table 14
Utility degree of alternatives

$U_{\Re}$	Y
U_1	10.62
U_2	13.60
U_3	6.03
U_4	9.70

Table 15
Utility degree of alternatives

$U_{\Re}$	Y
U_1	0.499
U_2	0.639
U_3	0.28
U_4	0.456

Step 9: Calculate the utility function for alternatives, represented as $\Gamma(\Psi\Re)$, using Equation (21).

Table 16
Utility degree of alternatives via equation (21)

$U_{\Re}$	Y
U_1	0.498
U_2	0.638
U_3	0.283
U_4	0.455

Step 10: Arrange the alternatives in order of their utility function values, aiming for each alternative to possess the highest achievable utility function value.

$$(U_2) > (U_1) > (U_4) > (U_3)$$

Table 17 shows the comparison of the ranking of this case study with both methods. Ranking of both methods is same which shows the exactness and impact of this method.

Table 17
Overall Ranking

$U_{\Re}$	WASPAS	Ranking	MARCOS	Ranking
U_1	0.25	4	0.498	2
U_2	0.50	1	0.638	1
U_3	0.30	3	0.283	4
U_4	0.38	2	0.455	3

Figure 4 shows the ranking of the alternatives from different methods.

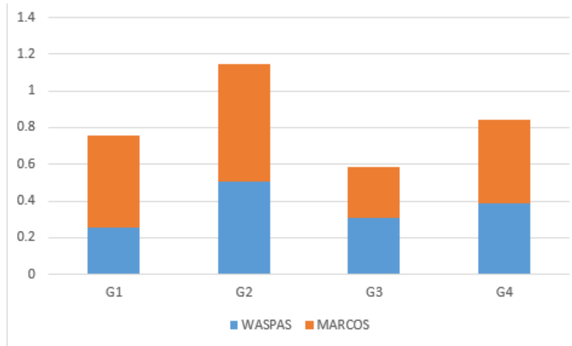

Fig. 4. Ranking of the alternatives

7. Conclusion

In this research, an innovative approach is unveiled, employing the synergies of WASPAS and MARCOS methodologies to effectively address the challenges posed by Complex Fuzzy Z-Numbers (CFZNs). This research illuminates AR's role in decision-making, specifically addressing complexities associated with Complex Fuzzy Z-Numbers, showcasing its expanding influence in nuanced decision scenarios. Based on the above analysis and the results obtained in Augmented Reality (AR) across various fields, the priority of the alternatives in the MARCOS method follows the sequence $(U_2) > (U_1) > (U_4) > (U_3)$. In the WASPAS method, the priority sequence is $(U_2) > (U_4) > (U_3) > (U_1)$.
The ensuing discussion examines the pivotal impact of this assessment:

- We systematically develop the set and properties of Complex Fuzzy Z-Numbers (CFZNs) along with their corresponding scoring and accuracy functions.
- We initiated the algorithm and implementation of the WASPAS technique in Complex Fuzzy Z-Numbers (CFZNs).
- Similarly,we commenced the algorithmic implementation of the MARCOS technique specifically tailored for Complex Fuzzy Z-Numbers (CFZNs).
- We demonstrated the Multiple Attribute Decision Making (MADM) strategy within the devised frameworks and applied these techniques in accordance with a case study, illustrated through a numerical example.
- We highlighted the superiority and dominance of the devised frameworks in comparison to the explored approaches.

In the future, our focus is on developing novel techniques like MMOORA, COPRAS, PROMETHEE for Complex Fuzzy Z-Numbers (CFZNs). We aim to apply these techniques in artificial intelligence, machine learning, game theory, neural networks, and clustering analysis to enhance the quality of the presented information.Despite challenges, there is optimism that the developed method has the potential to introduce a housebreaking approach to address decision-making complexities within various Complex Fuzzy Z-number (CFZN) environments.

Conflicts of Interest

The authors declare no conflicts of interest.

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