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# Enhancing Web Security Through Complex Cubic q-Rung Orthopair Fuzzy Information

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**ABSTRACT**

Over the past two decades, the evolution of web services has profoundly transformed communication and information technologies. As computer programs that facilitate data exchange and interoperability between applications via the Internet, web services play a pivotal role in enabling seamless interactions and information access. However, despite their advantages, these platforms also introduce significant security risks. This study leverages the novel framework of Complex Cubic q-Rung Orthopair Fuzzy Sets (CCuqROFS) to model and address these security threats effectively. Unlike conventional models, CCuqROFS provides a robust structure that incorporates membership (M) and non-membership (NM) degrees, offering a more precise representation of uncertainty. Through illustrative examples, we introduce key concepts such as Complex Cubic q-Rung Orthopair Fuzzy Relations (CCuqROFR), the Cartesian product of CCuqROFSs, and various forms of CCuqROFRs. For the first time in fuzzy set theory, this work systematically examines the relationships between different security threats and countermeasures in web services. The proposed methodologies demonstrate how robust security measures can mitigate the impact of these threats. Finally, a comparative analysis underscores the advantages of our strategies, providing valuable insights for enhancing web service security.

## 1. Introduction

Uncertainty theory, a branch of mathematics, is founded on the axioms of normalization, monotonicity, duality, countable subadditivity, and product measures. Alongside uncertainty, various mathematical frameworks such as probability theory, possibility theory, fuzzy logic, feasibility, and believability are used to assess the likelihood of events. Uncertainty arises from insufficient information about a situation or event, making it challenging to plan for or predict future outcomes, a state that is often discomforting, as most individuals prefer structure and routine. Zadeh [1] introduced the concept of fuzzy sets to address ambiguity and uncertainty in a mathematical context. Unlike classical binary set theory, fuzzy set theory allows elements to have a degree of membership, providing a more flexible and nuanced approach to classification. Klir and Yuan [2] comprehensively

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review fuzzy set theory and its applications, including fuzzy logic, control systems, and decision-making. Dubois and Prade [3] explore the connections between fuzzy sets and other theoretical frameworks such as statistical inference and probability theory. Buckley [4] investigated the concept of fuzzy probability distributions as a method for modeling uncertainty in statistical data. Zimmermann *et al.,* [5] offer an overview of fuzzy set theory and its applications across management, engineering, and scientific research. More recently, Jun *et al.,* [6] introduced the concept of the cubic set, further expanding the theoretical landscape for handling complex uncertainties.

A unique type of fuzzy set, called a cubic fuzzy set (CuFS), has a cubic M-function. A cubic linear function defines the M function in a CuFS, which accepts values between 0 and 1. Mehmood *et al.,* [7] investigated the concept of cubic hesitant fuzzy sets and their applications to multi-criteria decision-making. Ramot *et al.,* [8] were the ones who initially developed the concept of a complex fuzzy set (CFS). The CFS model's capability is highly dependent on the CFS phase term. This phrase distinguishes a CFS theory from all other published CFS theories. Complex fuzzy sets perform better at handling imperfect and reasonable features than time-periodic events in illustrating a variety of phenomena. A distance metric for complex fuzzy sets was presented by Wang *et al.,* [9] and is based on the modulus of the difference between two complex fuzzy sets. Complex fuzzy sets are mappings from a universe of discourse to the unit circle in the complex plane and were first developed by Tamir *et al.,* [10]. The manipulation and comparison of intricate fuzzy sets are made possible by the definitions of fundamental operations and distance measures put out by Wang and Zhang [11]. As his original publication presented the idea of intuitionistic fuzzy sets as a generalization of fuzzy sets, Atanassov [12] is regarded as the creator of intuitionistic fuzzy sets. Particularly in the fields of possibility theory and fuzzy logic, Dixit and Jain [13] contributions to the development of intuitionistic fuzzy sets have been significant. In their detailed study of intuitionistic fuzzy sets, Szmidt and Kacprzyk [14] covered the theoretical underpinnings, applications in diverse disciplines, and extensions and generalizations of these sets.

Szmidt and Kacprzyk [15] have extensively studied intuitionistic fuzzy sets and their applications in decision-making, and their survey paper offers a helpful overview of the subject. Complex intuitionistic fuzzy sets (CIFS) were introduced by Alkouri *et al.,* [16], who also described some of their features and operations. Garg and Kumar [17], who also describe some of their benefits, compare CIFS to other fuzzy sets. Complex intuitionistic fuzzy sets (CIFS), among other fuzzy sets, have been used in classification, clustering, and pattern recognition, according to Zimmermann's [18] discussion. Additionally, Zimmermann [18] offers a number of CIFS real-world applications in various fields. Garg and Rani [19] discussed the various methods for using CIFS to solve MCDM problems and presented some CIFS applications in this field. Alkouri and Salleh [20] proposed a complex intuitionistic fuzzy set (CIFS). The sum of the degrees of M and NM in the complex plane is constrained by CIFS to the unit disc. The CIFR, which was used for cybersecurity and cybercrime in the oil and gas industry, is provided by Jan *et al.,* [21]. Garg and Rani [22] developed the idea of an interval-valued complex intuitionistic fuzzy set (IVCIFS). Nasir *et al*., [23] developed the concept of an interval-valued complex intuitionistic fuzzy relation (IVCIFR) based on the relationship between two or more IVCFS. Garg and Kaur [24] have made significant contributions to the study of fuzzy sets and their extensions.

In addition, he proposed the concept of complex cubic intuitionistic fuzzy sets and illustrated how they might be used to solve classification and decision-making problems. Chinnadurai *et al*., [25] reported complex cubic intuitionistic fuzzy sets, along with their method and a comparison with other approaches. Additionally, they offer a case study to illustrate the value of their strategy. Mehmood and Liu [26] have studied complex cubic intuitionistic fuzzy sets for use in image processing. They also provided an explanation of their methodology and examples of its application to edge detection and image segmentation. Garg [27] provided an overview of Pythagorean fuzzy sets and their characteristics, along with various applications in decision-making, clustering, and image processing. When weight information is lacking, Wang and Feng [28] developed a Pythagorean fuzzy set-based method for multiple-attribute decision-making. Abdullah and [29] presented a Pythagorean fuzzy set-based multi-criteria decision-making technique utilising fuzzy preference relations. Muhammad *et al.,* [30] proposed a novel method for Pythagorean fuzzy decision-making based on conventional grey relational analysis. Yager [31] provides a study of Pythagorean fuzzy sets, including their definition, properties, applications across different domains, and operations.

Complex Pythagorean fuzzy sets and their fundamental operations were thoroughly explained by Ahmed and Mustafa [32], and examples of their use in decision-making were presented. Ejegwa [33] provided a thorough discussion of complex Pythagorean fuzzy sets and their fundamental principles, along with illustrations of their use in medical diagnosis. Ma *et al.,* [34], who have written extensively on the subject, studied experts in group decision-making. Significant advancements in the creation of complex cubic Pythagorean fuzzy sets and their use in decision-making have been made by Abbas *et al.,* [35] and Akram and Naz [36]. A novel decision-making technique based on complex cubic Pythagorean fuzzy sets was proposed by Saeed *et al.,* [37] and used to solve a practical issue. For complex cubic Pythagorean fuzzy sets, Khan *et al.,* [38] developed several new aggregation methods and illustrated their utility in decision-making. Ali [39] gave examples of the application of q-rung orthopair fuzzy sets in decision-making and pattern recognition. A variety of q-rung orthopair fuzzy graph types, including directed, weighted, and full graphs, were defined by Akram *et al.,* [40], along with examples of their construction and analysis.

The clustering approach was studied by Garg [41], who also provided a case study of how it was used to examine a dataset of customer transactions. A numerical example was given by Naz *et al.,* [42] to illustrate the viability of the suggested strategy. Using CQROFSs, Wang *et al.,* [43] suggested a decision-making procedure and contrasted it with other approaches already in use. Chu *et al.,* [44] conducted a study to show how well the suggested model works. A novel approach to multiple attribute group decision-making (MAGDM) using CQROFSs was proposed by Wang *et al.,* [45]. Zhang *et al.,* [46] investigated multi-attribute group decision-making, including the idea of cubic q-Rung orthopair fuzzy Heronian mean operators. Ali *et al.,* [47] studied the use of complex q-rung orthopair fuzzy sets.

This work introduced the unique ideas of the cubic q-rung orthopair fuzzy set (CCuqROFS) and CCuqROFRs. The Cartesian product of two or more CCuqROFSs is also considered. Moreover, several CCuqROFR types, including reflexive, irreflexive, symmetric, antisymmetric, transitive, equivalence classes, and many more, are specified. Each definition has been defined together with the applicable examples and CCuqROFRs findings. The CFR, CCuFR, CIFR, CCuIFR, CPyFR, CCuPyFR, and CqROFR are all upgraded versions of one another. In the CCuqROFR, the discussion covers both stages, M and NM. The multidimensional structure, described in terms of both amplitude and phase, is the CCuqROFR. The time period or periodicity is described using the phase word. In this paper, we investigate the connection between web services security and specific threats. The CCuqROFR is therefore a superior framework to employ because it can account for both the situation's positive and negative long-term impacts. The ideal security measures are chosen using the CCuqROFR ideas. CCuqROFRs' original concept considers both the present and the future. Additionally, this framework can be expanded to various fuzzy set theory models for use across a range of other industries, including computer science, medicine, sports, economics, statistics, and information technology. Table 1 shows the organization of this article.

The arrangements of this paper are as follows: Section 1 consists of an introduction; Section 2 discusses some previously defined concepts. Section 3 presents the new notions of complex q-rung orthopair fuzzy relations and associated types with examples. Section 4 proposes the application involving communication and web services security. Section 5 carries out a comparative analysis of the proposed structure with the existing frameworks. In Section 6, the article is concluded.

## 2. Preliminaries

In this section, we define some basic definitions of CuFS, CuFR, CCuFS, CIFS, CIVIFS, and CqROFS.

Definition 1 [7]. For a universal set Ù, CuFS Ҡ on Ù is defined as

$$Ҡ = \{ţ, (F(ţ)), (G(ţ)): ţ \in Ù\}$$

Where $(ţ) = \{(ţ, [p^{-}(ţ), p^{+}(ţ)]): ţ \in Ù\}$ represents the IVFS defined on Ù and $G(ţ) = \{(ţ, p(ţ)): ţ \in Ù\}$ represents the FS. Moreover, $p^{-}(ţ), p^{+}(ţ)$ are called the left and right degrees of M respectively, and $p(ţ)$ is called M degree. Such that $p(ţ): Ù \rightarrow [0,1]$, and also $0 \leq p^{-}(ţ) \leq p^{+}(ţ) \leq 1$. Therefore, a CuFS can be written as

$$Ҡ = \{ţ, ([p^{-}(ţ), p^{+}(ţ)]), (p(ţ)): ţ \in Ù\}.$$

Definition 2 [7]. Let us take two CuFSs in a non-empty set Ù,

$$j = \left\{ţ, \left([\alpha^{-}_{j(p)}(ţ), \alpha^{+}_{j(p)}(ţ)]\right), \left(\alpha_{j(p)}(ţ)\right): ţ \in Ù\right\},$$

$$\mathbb{H} = \left\{ḟ, \left([\alpha^{-}_{\mathbb{H}(p)}(ḟ), \alpha^{+}_{\mathbb{H}(p)}(ḟ)], \left(\alpha_{\mathbb{H}(p)}(ḟ)\right)\right): ḟ \in Ù\right\}.$$

Then their Cartesian product is,

$$j \times \mathbb{H} = \left\{(ţ, ḟ), \left([\alpha^{-}_{j\times\mathbb{H}(p)}(ţ, ḟ), \alpha^{+}_{j\times\mathbb{H}(p)}(ţ, ḟ)]\right), \left(\alpha_{j\times\mathbb{H}(p)}(ţ, ḟ)\right)\right\}$$

where,

$$\alpha^{-}_{j\times\mathbb{H}(p)}(ţ, ḟ) = min\{\alpha^{-}_{j(p)}(ţ), \alpha^{-}_{\mathbb{H}(p)}(ḟ)\},$$
$$\alpha^{+}_{j\times\mathbb{H}(p)}(ţ, ḟ) = min\{\alpha^{+}_{j(p)}(ţ), \alpha^{+}_{\mathbb{H}(p)}(ḟ)\},$$
$$\alpha^{-}_{j\times\mathbb{H}(p)}(ţ, ḟ) = min\{\alpha_{j(p)}(ţ), \alpha_{\mathbb{H}(p)}(ḟ)\}.$$

The subset of the Cartesian product of two CuFSs is called a cubic fuzzy relation (CuFR).

Example 1: Let two CuFSs j and $\mathbb{H}$ on Ù is defined as,

$$j = \{(ţ_1, [0.3,0.5]), (0.4), (ţ_2, [0.2,0.6]), (0.3)\},$$
$$\mathbb{H} = \{(ḟ_1, [0.2,0.5]), (0.3)(ḟ_2, [0.3,0.7]), (0.2)\}$$

Then their Cartesian product is,

$$j \times \mathbb{H} = \begin{Bmatrix} \left((ţ_1, ḟ_1), [0.2,0.5]\right), (0.3), \left((ţ_1, ḟ_2), [0.3,0.5]\right), (0.2) \\ \left((ţ_2, ḟ_1), [0.2,0.5]\right), (0.3), \left((ţ_2, ḟ_2), [0.3,0.5]\right), (0.2) \end{Bmatrix}$$

The subset of $j \times \mathbb{H}$ is called relation

$$Ŗ = \left\{\left((ţ_1, ḟ_1), [0.2,0.5]\right), (0.3), \left((ţ_2, ḟ_1), [0.2,0.5]\right), (0.3)\right\}$$

Definition 3 [24]. For a universal set Ù, a complex (CCuFS) Ҡ, on Ù is defined as

$$Ҡ = \{ţ, ([p^{-}_{c}(ţ), p^{+}_{c}(ţ)]), (, p_c(ţ)): ţ \in Ù\},$$

Where, $p^{-}_{c}(ţ), p^{+}_{c}(ţ)$ represents the left and right degrees of M, respectively. Thus, $p^{-}_{c}(ţ) = \alpha^{-}_{p}(ţ)e^{\beta^{-}_{p}(ţ)лi}, p^{+}_{c}(ţ) = \alpha^{+}_{p}(ţ)e^{\beta^{+}_{p}(ţ)лi}$ such that $0 \leq |p^{-}_{c}(ţ)| \leq |p^{+}_{c}(ţ)| \leq 1$, and $p_c(ţ) = \alpha_p(ţ)e^{\beta_p(ţ)лi}$. Moreover, $\alpha^{-}_{p}, \alpha^{+}_{p}, \alpha_p \epsilon [0,1]$ are called the amplitude term of M degree, and satisfy the following conditions $\alpha^{-}_{p} \leq \alpha^{+}_{p}$, and $\beta^{-}_{p}, \beta^{+}_{p}, \beta_p \epsilon [0,1]$ are called the phase term of degree of M, and satisfy the following condition, $\beta^{-}_{p} \leq \beta^{+}_{p}$.

Definition 4 [16]. For a universal set Ù, a complex intuitionistic fuzzy set (CIFS) Ҡ on Ù is defined as

$$Ҡ = \{ţ, (p_c(ţ), n_c(ţ)): ţ \in Ù\},$$

Where $p_c(ţ), n_c(ţ)$ is a complex valued mapping i.e., $p_c, n_c: Ù \longrightarrow \{\xi: \xi \in Ҡ\}$ and $|\xi| \leq 1$, which represents the M and NM degrees of the CIFS, respectively. Where $C$ is the set of the complex numbers. Thus, $p_c(ţ) = \alpha_p(ţ)e^{\beta_p(ţ)лi}$ and $n_c(ţ) = \alpha_n(ţ)e^{\beta_n(ţ)лi}$. Moreover, $\alpha_p, \alpha_n \in [0,1]$ are called the amplitude terms of the M and NM degrees respectively, and $\beta_p, \beta_n \in [0,1]$ are called the phase terms of the M and NM degrees respectively, and satisfy the following conditions,

$$0 \leq \alpha_p(ţ) + \alpha_n(ţ) \leq 1 \text{ and } 0 \leq \beta_p(ţ) + \beta_n(ţ) \leq 1$$

Definition 5 [34]. For a universal set Ù, a complex interval valued intuitionistic fuzzy set (CIVIFS) Ҡ on Ù is defined as,

$$Ҡ = \{ţ, (p_c(ţ), n_c(ţ)) : ţ \in Ù\},$$

Where $p_c(ţ) = [p_c^-(ţ), p_c^+(ţ)], n_c(ţ) = [n_c^-(ţ), n_c^+(ţ)]$, and $p_c^-(ţ), p_c^+(ţ)$ are called the left and right M degrees, while $n_c^-(ţ), n_c^+(ţ)$ are called the left and right NM degrees.

$p_c^-(ţ) = \alpha_p^-(ţ)e^{\beta_p^-(ţ)лi}, p_c^+(ţ) = \alpha_p^+(ţ)e^{\beta_p^+(ţ)лi}, n_c^-(ţ) = \alpha_n^-(ţ)e^{\beta_n^-(ţ)лi}, n_c^+(ţ) = \alpha_n^+(ţ)e^{\beta_n^+(ţ)лi}$, is given as,

$$0 \leq |p_c^+(ţ)| + |n_c^+(ţ)| \leq 1$$

Furthermore, CIVIFS can be written as,

$$Ҡ = \left\{ţ, \left[\alpha_p^-(ţ), \alpha_p^+(ţ)\right]e^{[\beta_p^-(ţ), \beta_p^+(ţ)]лi}, [\alpha_n^-(ţ), \alpha_n^+(ţ)]e^{[\beta_n^-(ţ), \beta_n^+(ţ)]лi}\right\}.$$

Definition 6 [47]. For a universal set Ù, a complex q-rung orthopair fuzzy set (CqROFS) Ҡ on Ù is defined as,

$$Ҡ = \{ţ, (p_c(ţ), n_c(ţ)) : ţ \in Ù\},$$

Where, $p_c(ţ), n_c(ţ)$ is a complex valued mapping i.e., $p_c, n_c : Ù \longrightarrow \{\xi : \xi \in Ҡ\}$ and $|\xi| \leq 1$, which represents the M and NM degrees of the qROFS, respectively. Where $C$ is the set of the complex numbers. Thus, $p_c(ţ) = \alpha_p(ţ)e^{\beta_p(ţ)лi}$ and $n_c(ţ) = \alpha_n(ţ)e^{\beta_n(ţ)лi}$. Moreover, $\alpha_p, \alpha_n \in [0,1]$ are called the amplitude terms of the M and NM degrees respectively, and $\beta_p, \beta_n \in [0,1]$ are called the phase terms of the M and NM degrees respectively, and satisfy the following conditions,

$0 \leq \left(\alpha_p(ţ)\right)^n + \left(\alpha_n(ţ)\right)^n \leq 1$ and $0 \leq \left(\beta_p(ţ)\right)^n + \left(\beta_n(ţ)\right)^n \leq 1$ where $n \in N$.

## 3. Main Results

This section introduces the novel concepts of cubic q-rung ortho-pair fuzzy set (CuqROFS), complex cubic q-rung ortho-pair fuzzy set (CCuqROFS), and the Cartesian product of two CCuqROFS, and its types.

Definition 7. For a universal set Ù, a CuqROFS Ҡ on Ù is defined as

$$Ҡ = \{ţ, (F(ţ)), (G(ţ)) : ţ \in Ù\}$$

Where, $F(ţ) = \{(ţ, [p^-(ţ), p^+(ţ)], [n^-(ţ), n^+(ţ)]) : ţ \in Ù\}$ embodies the interval-valued qROFS and $G(ţ) = \{(ţ, p(ţ), n(ţ)) : ţ \in Ù\}$ embodies the qROFS of Ù. Where $p^-(ţ), p^+(ţ)$ represents the left and right degrees of M and $n^-(ţ), n^+(ţ)$ represents the left and right degrees of NMWhere $p(ţ), n(ţ)$ are called the M, and NM degrees, respectively. Such that $0 \leq p^-(ţ) \leq p^+(ţ) \leq 1$, and $0 \leq n^-(ţ) \leq n^+(ţ) \leq 1$ furthermore, $p(ţ), n(ţ) \in [0,1]$, on condition that $0 \leq |p(ţ)|^n, |n(ţ)|^n \leq 1, n \in N$.

Definition 8. For a universal set Ù, a CCuqROFS Ҡ on Ù is defined as

$$Ҡ = \left\{ţ, \left(([p_c^-(ţ), p_c^+(ţ)], [n_c^-(ţ), n_c^+(ţ)]), (p_c(ţ), n_c(ţ)) : ţ \in Ù\right)\right\}$$

Where $p_c^-(ţ), p_c^+(ţ), n_c^-(ţ), n_c^+(ţ)$ are called the left and right M and NM degrees, respectively while $p_c(ţ), n_c(ţ)$ are called M and NM degrees defined as $p_c(ţ) : Ù \to \{\xi_p : \xi_p \in Ҡ \text{ and } |\xi_p| \leq 1\}$, $n_c(ţ) : Ù \to \{\xi_n : \xi_n \in Ҡ \text{ and } |\xi_n| \leq 1\}$. Ҡ is the set of complex number and the complex numbers can be written as $p_c(ţ) = \alpha_p(ţ)e^{\beta_p(ţ)лi}$, $n_c(ţ) = \alpha_n(ţ)e^{\beta_n(ţ)лi}$. Therefore, CCuqROFS can be expressed as

$$Ҡ = \left\{ţ, \left(\begin{pmatrix}[\alpha_p^-(ţ), \alpha_p^+(ţ)]e^{[\beta_p^-(ţ), \beta_p^+(ţ)]лi}, \\ [\alpha_n^-(ţ), \alpha_n^+(ţ)]e^{[\beta_n^-(ţ), \beta_n^+(ţ)]лi}\end{pmatrix}, \begin{pmatrix}\alpha_p(ţ)e^{\beta_p(ţ)лi}, \\ \alpha_n(ţ)e^{\beta_n(ţ)лi}\end{pmatrix}\right) : ţ \in Ù\right\}.$$

Moreover, $\alpha_p^-(\text{ţ}), \alpha_p^+(\text{ţ}), \alpha_p(\text{ţ}), \beta_p^-(\text{ţ}), \beta_p^+(\text{ţ}), \beta_p(\text{ţ}): \grave{U} \to [0,1]$ are the elements of M degree, $\alpha_n^-(\text{ţ}), \alpha_n^+(\text{ţ}), \alpha_n(\text{ţ}), \beta_n^-(\text{ţ}), \beta_n^+(\text{ţ}), \beta_n(\text{ţ}): \grave{U} \to [0,1]$ are the elements of NM degree, such that $0 \leq \left(\alpha_p(\text{ţ})\right)^n + \left(\alpha_n(\text{ţ})\right)^n \leq 1$ and $0 \leq \left(\beta_p(\text{ţ})\right)^n + \left(\beta_n(\text{ţ})\right)^n \leq 1 \; n \in N$.

Definition 9. For a universal set Ù, let us take two CCuqROFSs,

$$\text{j} = \left\{ \text{ţ}, \left( \begin{pmatrix} [\alpha_{\text{j}(p)}^-(\text{ţ}), \alpha_{\text{j}(p)}^+(\text{ţ})] e^{[\beta_{\text{j}(p)}^-(\text{ţ}), \beta_{\text{j}(p)}^+(\text{ţ})]\pi i}, \\ [\alpha_{\text{j}(n)}^-(\text{ţ}), \alpha_{\text{j}(n)}^+(\text{ţ})] e^{[\beta_{\text{j}(n)}^-(\text{ţ}), \beta_{\text{j}(n)}^+(\text{ţ})]\pi i} \end{pmatrix}, \begin{pmatrix} \alpha_{\text{j}(p)}(\text{ţ}) e^{\beta_{\text{j}(p)}(\text{ţ})\pi i}, \\ \alpha_{\text{j}(n)}(\text{ţ}) e^{\beta_{\text{j}(n)}(\text{ţ})\pi i} \end{pmatrix} \right) : \text{ţ} \in \grave{U} \right\} \text{ and,}$$

$$\mathbb{H} = \left\{ \dot{f}, \left( \begin{pmatrix} [\alpha_{\mathbb{H}(p)}^-(\dot{f}), \alpha_{\mathbb{H}(p)}^+(\dot{f})] e^{[\beta_{\mathbb{H}(p)}^-(\dot{f}), \beta_{\mathbb{H}(p)}^+(\dot{f})]\pi i}, \\ [\alpha_{\mathbb{H}(n)}^-(\dot{f}), \alpha_{\mathbb{H}(n)}^+(\dot{f})] e^{[\beta_{\mathbb{H}(n)}^-(\dot{f}), \beta_{\mathbb{H}(n)}^+(\dot{f})]\pi i} \end{pmatrix}, \begin{pmatrix} \alpha_{\mathbb{H}(p)}(\dot{f}) e^{\beta_{\mathbb{H}(p)}(\dot{f})\pi i}, \\ \alpha_{\mathbb{H}(n)}(\dot{f}) e^{\beta_{\mathbb{H}(n)}(\dot{f})\pi i} \end{pmatrix} \right) : \dot{f} \in \grave{U} \right\}$$

Then their Cartesian product is defined as

$$\text{j} \times \mathbb{H} = \left\{ (\text{ţ}, \dot{f}), \left( \begin{matrix} \begin{pmatrix} [\alpha_{\text{j}\times\mathbb{H}(p)}^-(\text{ţ}, \dot{f}), \alpha_{\text{j}\times\mathbb{H}(p)}^+(\text{ţ}, \dot{f})] e^{[\beta_{\text{j}\times\mathbb{H}(p)}^-(\text{ţ},\dot{f}), \beta_{\text{j}\times\mathbb{H}(p)}^+(\text{ţ},\dot{f})]\pi i}, \\ [\alpha_{\text{j}\times\mathbb{H}(n)}^-(\text{ţ}, \dot{f}), \alpha_{\text{j}\times\mathbb{H}(n)}^+(\text{ţ}, \dot{f})] e^{[\beta_{\text{j}\times\mathbb{H}(n)}^-(\text{ţ},\dot{f}), \beta_{\text{j}\times\mathbb{H}(n)}^+(\text{ţ},\dot{f})]\pi i} \end{pmatrix}, \\ \begin{pmatrix} \alpha_{\text{j}\times\mathbb{H}(p)}(\text{ţ}, \dot{f}) e^{\beta_{\text{j}\times\mathbb{H}(p)}(\text{ţ},\dot{f})\pi i}, \\ \alpha_{\text{j}\times\mathbb{H}(n)}(\text{ţ}, \dot{f}) e^{\beta_{\text{j}\times\mathbb{H}(n)}(\text{ţ},\dot{f})\pi i} \end{pmatrix} \end{matrix} \right) : \text{ţ} \in \grave{U} \right\}$$

where,

$$\alpha_{\text{j}\times\mathbb{H}(p)}^-(\text{ţ}, \dot{f}) e^{\beta_{\text{j}\times\mathbb{H}(p)}^-(\text{ţ},\dot{f})} = min\{\alpha_{\text{j}(p)}^-(\text{ţ}), \alpha_{\mathbb{H}(p)}^-(\dot{f})\} e^{min\{\beta_{\text{j}(p)}^-(\text{ţ}), \beta_{\mathbb{H}(p)}^-(\dot{f})\}}$$
$$\alpha_{\text{j}\times\mathbb{H}(p)}^+(\text{ţ}, \dot{f}) e^{\beta_{\text{j}\times\mathbb{H}(p)}^+(\text{ţ},\dot{f})} = min\{\alpha_{\text{j}(p)}^+(\text{ţ}), \alpha_{\mathbb{H}(p)}^+(\dot{f})\} e^{min\{\beta_{\text{j}(p)}^+(\text{ţ}), \beta_{\mathbb{H}(p)}^+(\dot{f})\}}$$
$$\alpha_{\text{j}\times\mathbb{H}(n)}^-(\text{ţ}, \dot{f}) e^{\beta_{\text{j}\times\mathbb{H}(n)}^-(\text{ţ},\dot{f})} = max\{\alpha_{\text{j}(n)}^-(\text{ţ}), \alpha_{\mathbb{H}(n)}^-(\dot{f})\} e^{min\{\beta_{\text{j}(n)}^-(\text{ţ}), \beta_{\mathbb{H}(n)}^-(\dot{f})\}}$$
$$\alpha_{\text{j}\times\mathbb{H}(n)}^+(\text{ţ}, \dot{f}) e^{\beta_{\text{j}\times\mathbb{H}(n)}^+(\text{ţ},\dot{f})} = max\{\alpha_{\text{j}(n)}^+(\text{ţ}), \alpha_{\mathbb{H}(n)}^+(\dot{f})\} e^{min\{\beta_{\text{j}(n)}^+(\text{ţ}), \beta_{\mathbb{H}(n)}^+(\dot{f})\}}$$

Example 2: Let two CCuqROFSs j and $\mathbb{H}$ on Ù, is defined as for $n = 5$

$$\text{j} = \left\{ \begin{matrix} \begin{pmatrix} \text{ţ}_1, ([0.5,0.6]e^{[0.4,0.7]\pi i}, [0.4,0.7]e^{[0.3,0.8]\pi i}), \\ (0.5e^{(0.5)\pi i}, 0.5e^{(0.5)\pi i}) \end{pmatrix}, \\ \begin{pmatrix} \text{ţ}_2, ([0.4,0.5]e^{[0.4,0.6]\pi i}, [0.2,0.4]e^{[0.4,0.7]\pi i}), \\ (0.3e^{(0.4)\pi i}, 0.4e^{(0.6)\pi i}) \end{pmatrix} \end{matrix} \right\},$$

$$\mathbb{H} = \left\{ \begin{matrix} \begin{pmatrix} \dot{f}_1, ([0.3,0.6]e^{[0.2,0.5]\pi i}, [0.4,0.5]e^{[0.3,0.5]\pi i}), \\ (0.3e^{(0.6)\pi i}, 0.3e^{(0.7)\pi i}) \end{pmatrix}, \\ \begin{pmatrix} \dot{f}_2, ([0.2,0.7]e^{[0.5,0.6]\pi i}, [0.3,0.5]e^{[0.5,0.8]\pi i}), \\ (0.4e^{(0.5)\pi i}, 0.6e^{(0.7)\pi i}) \end{pmatrix} \end{matrix} \right\}.$$

Then, their Cartesian product is

$$\text{j} \times \mathbb{H} = \left\{ \begin{matrix} \begin{pmatrix} (\text{ţ}_1, \dot{f}_1), ([0.3,0.6]e^{[0.2,0.5]\pi i}, [0.4,0.7]e^{[0.3,0.8]\pi i}), \\ (0.3e^{(0.5)\pi i}, 0.5e^{(0.7)\pi i}) \end{pmatrix}, \\ \begin{pmatrix} (\text{ţ}_1, \dot{f}_2), ([0.4,0.5]e^{[0.4,0.6]\pi i}, [0.2,0.4]e^{[0.4,0.7]\pi i}), \\ (0.3e^{(0.4)\pi i}, 0.4e^{(0.6)\pi i}) \end{pmatrix}, \\ \begin{pmatrix} (\text{ţ}_2, \dot{f}_1), ([0.3,0.5]e^{[0.2,0.5]\pi i}, [0.4,0.5]e^{[0.4,0.7]\pi i}), \\ (0.3e^{(0.4)\pi i}, 0.4e^{(0.7)\pi i}) \end{pmatrix}, \\ \begin{pmatrix} (\text{ţ}_2, \dot{f}_2), ([0.2,0.5]e^{[0.4,0.6]\pi i}, [0.3,0.5]e^{[0.5,0.8]\pi i}), \\ (0.3e^{(0.4)\pi i}, 0.6e^{(0.7)\pi i}) \end{pmatrix} \end{matrix} \right\}$$

Definition 10. A CCuqROFR denoted by $\bar{\text{Ŗ}}$ is a subset of Cartesian product of two CCuqROFSs.

Example 3: Choose a relation from example 1 is given as

$$\bar{\text{Ṛ}} = \left\{\begin{matrix}\begin{pmatrix}(\text{ţ}_1, \dot{\text{f}}_2), ([0.4,0.5]e^{[0.4,0.6]\text{л}i}, [0.2,0.4]e^{[0.4,0.7]\text{л}i}), \\ (0.3e^{(0.4)\text{л}i}, 0.4e^{(0.6)\text{л}i})\end{pmatrix}, \\ \begin{pmatrix}(\text{ţ}_2, \dot{\text{f}}_2), ([0.2,0.5]e^{[0.4,0.6]\text{л}i}, [0.3,0.5]e^{[0.5,0.8]\text{л}i}), \\ (0.3e^{(0.4)\text{л}i}, 0.6e^{(0.7)\text{л}i})\end{pmatrix}\end{matrix}\right\}$$

Definition 11. Let $\bar{\text{Ṛ}}$ be a CCuqROFR on a CCuqROFS Ҟ

$$\left\{\text{ţ}, \left(\begin{pmatrix}[\alpha_{(p)}^{-}(\text{ţ}), \alpha_{(p)}^{+}(\text{ţ})]e^{[\beta_{(p)}^{-}(\text{ţ}),\beta_{(p)}^{+}(\text{ţ})]\text{л}i}, \\ [\alpha_{(n)}^{-}(\text{ţ}), \alpha_{(n)}^{+}(\text{ţ})]e^{[\beta_{(n)}^{-}(\text{ţ}),\beta_{(n)}^{+}(\text{ţ})]\text{л}i}\end{pmatrix}, \begin{pmatrix}\alpha_{(p)}(\text{ţ})e^{\beta_{(p)}(\text{ţ})\text{л}i}, \\ \alpha_{(n)}(\text{ţ})e^{\beta_{(n)}(\text{ţ})\text{л}i}\end{pmatrix}\right)\right\},$$

$$\left\{\dot{\text{f}}, \left(\begin{pmatrix}[\alpha_{(p)}^{-}(\dot{\text{f}}), \alpha_{(p)}^{+}(\dot{\text{f}})]e^{[\beta_{(p)}^{-}(\dot{\text{f}}),\beta_{(p)}^{+}(\dot{\text{f}})]\text{л}i}, \\ [\alpha_{(n)}^{-}(\dot{\text{f}}), \alpha_{(n)}^{+}(\dot{\text{f}})]e^{[\beta_{(n)}^{-}(\dot{\text{f}}),\beta_{(n)}^{+}(\dot{\text{f}})]\text{л}i}\end{pmatrix}, \begin{pmatrix}\alpha_{(p)}(\dot{\text{f}})e^{\beta_{(p)}(\dot{\text{f}})\text{л}i}, \\ \alpha_{(n)}(\dot{\text{f}})e^{\beta_{(n)}(\dot{\text{f}})\text{л}i}\end{pmatrix}\right)\right\},$$

$$\left\{\acute{\text{k}}, \left(\begin{pmatrix}[\alpha_{(p)}^{-}(\acute{\text{k}}), \alpha_{(p)}^{+}(\acute{\text{k}})]e^{[\beta_{(p)}^{-}(\acute{\text{k}}),\beta_{(p)}^{+}(\acute{\text{k}})]\text{л}i}, \\ [\alpha_{(n)}^{-}(\acute{\text{k}}), \alpha_{(n)}^{+}(\acute{\text{k}})]e^{[\beta_{(n)}^{-}(\acute{\text{k}}),\beta_{(n)}^{+}(\acute{\text{k}})]\text{л}i}\end{pmatrix}, \begin{pmatrix}\alpha_{(p)}(\acute{\text{k}})e^{\beta_{(p)}(\acute{\text{k}})\text{л}i}, \\ \alpha_{(n)}(\acute{\text{k}})e^{\beta_{(n)}(\acute{\text{k}})\text{л}i}\end{pmatrix}\right)\right\} \in \text{Ҡ}$$

(i) A CCuqROFR $\bar{\text{Ṛ}}$ is called reflexive, if

$$\forall\left\{\text{ţ}, \left(\begin{pmatrix}[\alpha_{(p)}^{-}(\text{ţ}), \alpha_{(p)}^{+}(\text{ţ})]e^{[\beta_{(p)}^{-}(\text{ţ}),\beta_{(p)}^{+}(\text{ţ})]\text{л}i}, \\ [\alpha_{(n)}^{-}(\text{ţ}), \alpha_{(n)}^{+}(\text{ţ})]e^{[\beta_{(n)}^{-}(\text{ţ}),\beta_{(n)}^{+}(\text{ţ})]\text{л}i}\end{pmatrix}, \begin{pmatrix}\alpha_{(p)}(\text{ţ})e^{\beta_{(p)}(\text{ţ})\text{л}i}, \\ \alpha_{(n)}(\text{ţ})e^{\beta_{(n)}(\text{ţ})\text{л}i}\end{pmatrix}\right)\right\} \in \text{Ҡ}$$

Implies,

$$\left\{(\text{ţ},\text{ţ}), \left(\begin{pmatrix}[\alpha_{(p)}^{-}(\text{ţ},\text{ţ}), \alpha_{(p)}^{+}(\text{ţ},\text{ţ})]e^{[\beta_{(p)}^{-}(\text{ţ},\text{ţ}),\beta_{(p)}^{+}(\text{ţ},\text{ţ})]\text{л}i}, \\ [\alpha_{(n)}^{-}(\text{ţ},\text{ţ}), \alpha_{(n)}^{+}(\text{ţ},\text{ţ})]e^{[\beta_{(n)}^{-}(\text{ţ},\text{ţ}),\beta_{(n)}^{+}(\text{ţ},\text{ţ})]\text{л}i}\end{pmatrix}, \begin{pmatrix}\alpha_{(p)}(\text{ţ},\text{ţ})e^{\beta_{(p)}(\text{ţ},\text{ţ})\text{л}i}, \\ \alpha_{(n)}(\text{ţ},\text{ţ})e^{\beta_{(n)}(\text{ţ},\text{ţ})\text{л}i}\end{pmatrix}\right)\right\} \in \bar{\text{Ṛ}}.$$

(ii) A CCuqROFR $\bar{\text{Ṛ}}$ is called symmetric, if

$$\left\{(\text{ţ},\dot{\text{f}}), \left(\begin{pmatrix}[\alpha_{(p)}^{-}(\text{ţ},\dot{\text{f}}), \alpha_{(p)}^{+}(\text{ţ},\dot{\text{f}})]e^{[\beta_{(p)}^{-}(\text{ţ},\dot{\text{f}}),\beta_{(p)}^{+}(\text{ţ},\dot{\text{f}})]\text{л}i}, \\ [\alpha_{(n)}^{-}(\text{ţ},\dot{\text{f}}), \alpha_{(n)}^{+}(\text{ţ},\dot{\text{f}})]e^{[\beta_{(n)}^{-}(\text{ţ},\dot{\text{f}}),\beta_{(n)}^{+}(\text{ţ},\dot{\text{f}})]\text{л}i}\end{pmatrix}, \begin{pmatrix}\alpha_{(p)}(\text{ţ},\dot{\text{f}})e^{\beta_{(p)}(\text{ţ},\dot{\text{f}})\text{л}i}, \\ \alpha_{(n)}(\text{ţ},\dot{\text{f}})e^{\beta_{(n)}(\text{ţ},\dot{\text{f}})\text{л}i}\end{pmatrix}\right)\right\} \in \bar{\text{Ṛ}}, \text{ And,}$$

$$\left\{(\dot{\text{f}},\text{ţ}), \left(\begin{pmatrix}[\alpha_{(p)}^{-}(\dot{\text{f}},\text{ţ}), \alpha_{(p)}^{+}(\dot{\text{f}},\text{ţ})]e^{[\beta_{(p)}^{-}(\dot{\text{f}},\text{ţ}),\beta_{(p)}^{+}(\dot{\text{f}},\text{ţ})]\text{л}i}, \\ [\alpha_{(n)}^{-}(\dot{\text{f}},\text{ţ}), \alpha_{(n)}^{+}(\dot{\text{f}},\text{ţ})]e^{[\beta_{(n)}^{-}(\dot{\text{f}},\text{ţ}),\beta_{(n)}^{+}(\dot{\text{f}},\text{ţ})]\text{л}i}\end{pmatrix}, \begin{pmatrix}\alpha_{(p)}(\dot{\text{f}},\text{ţ})e^{\beta_{(p)}(\dot{\text{f}},\text{ţ})\text{л}i}, \\ \alpha_{(n)}(\dot{\text{f}},\text{ţ})e^{\beta_{(n)}(\dot{\text{f}},\text{ţ})\text{л}i}\end{pmatrix}\right)\right\} \in \bar{\text{Ṛ}}.$$

(iii) A CCuqROFR $\bar{\text{Ṛ}}$ is called transitive, if

$$\left\{(\text{ţ},\dot{\text{f}}), \left(\begin{pmatrix}[\alpha_{(p)}^{-}(\text{ţ},\dot{\text{f}}), \alpha_{(p)}^{+}(\text{ţ},\dot{\text{f}})]e^{[\beta_{(p)}^{-}(\text{ţ},\dot{\text{f}}),\beta_{(p)}^{+}(\text{ţ},\dot{\text{f}})]\text{л}i}, \\ [\alpha_{(n)}^{-}(\text{ţ},\dot{\text{f}}), \alpha_{(n)}^{+}(\text{ţ},\dot{\text{f}})]e^{[\beta_{(n)}^{-}(\text{ţ},\dot{\text{f}}),\beta_{(n)}^{+}(\text{ţ},\dot{\text{f}})]\text{л}i}\end{pmatrix}, \begin{pmatrix}\alpha_{(p)}(\text{ţ},\dot{\text{f}})e^{\beta_{(p)}(\text{ţ},\dot{\text{f}})\text{л}i}, \\ \alpha_{(n)}(\text{ţ},\dot{\text{f}})e^{\beta_{(n)}(\text{ţ},\dot{\text{f}})\text{л}i}\end{pmatrix}\right)\right\} \in \bar{\text{Ṛ}}, \text{ and}$$

$$\left\{(\dot{\text{f}},\acute{\text{k}}), \left(\begin{pmatrix}[\alpha_{(p)}^{-}(\dot{\text{f}},\acute{\text{k}}), \alpha_{(p)}^{+}(\dot{\text{f}},\acute{\text{k}})]e^{[\beta_{(p)}^{-}(\dot{\text{f}},\acute{\text{k}}),\beta_{(p)}^{+}(\dot{\text{f}},\acute{\text{k}})]\text{л}i}, \\ [\alpha_{(n)}^{-}(\dot{\text{f}},\acute{\text{k}}), \alpha_{(n)}^{+}(\dot{\text{f}},\acute{\text{k}})]e^{[\beta_{(n)}^{-}(\dot{\text{f}},\acute{\text{k}}),\beta_{(n)}^{+}(\dot{\text{f}},\acute{\text{k}})]\text{л}i}\end{pmatrix}, \begin{pmatrix}\alpha_{(p)}(\dot{\text{f}},\acute{\text{k}})e^{\beta_{(p)}(\dot{\text{f}},\acute{\text{k}})\text{л}i}, \\ \alpha_{(n)}(\dot{\text{f}},\acute{\text{k}})e^{\beta_{(n)}(\dot{\text{f}},\acute{\text{k}})\text{л}i}\end{pmatrix}\right)\right\} \in \bar{\text{Ṛ}}, \quad \text{implies}$$

$$\left\{(\acute{\text{k}},\text{ţ}), \left(\begin{pmatrix}[\alpha_{(p)}^{-}(\acute{\text{k}},\text{ţ}), \alpha_{(p)}^{+}(\acute{\text{k}},\text{ţ})]e^{[\beta_{(p)}^{-}(\acute{\text{k}},\text{ţ}),\beta_{(p)}^{+}(\acute{\text{k}},\text{ţ})]\text{л}i}, \\ [\alpha_{(n)}^{-}(\acute{\text{k}},\text{ţ}), \alpha_{(n)}^{+}(\acute{\text{k}},\text{ţ})]e^{[\beta_{(n)}^{-}(\acute{\text{k}},\text{ţ}),\beta_{(n)}^{+}(\acute{\text{k}},\text{ţ})]\text{л}i}\end{pmatrix}, \begin{pmatrix}\alpha_{(p)}(\acute{\text{k}},\text{ţ})e^{\beta_{(p)}(\acute{\text{k}},\text{ţ})\text{л}i}, \\ \alpha_{(n)}(\acute{\text{k}},\text{ţ})e^{\beta_{(n)}(\acute{\text{k}},\text{ţ})\text{л}i}\end{pmatrix}\right)\right\} \in \bar{\text{Ṛ}}.$$

(iv) A CCuqROFR $\bar{Ṛ}$ is called equivalence, if it is
- ✓ Reflexive;
- ✓ Symmetric;
- ✓ Transitive.

(v) A CCuqROFR $\bar{Ṛ}$ is called partial order, if it is
- ✓ Reflexive;
- ✓ Symmetric;
- ✓ Transitive.

(vi) A CCuqROFR $\bar{Ṛ}$ is called preorder, if it is
- ✓ Reflexive;
- ✓ Transitive.

(vii) A CCuqROFR $\bar{Ṛ}$ is called strict order, if it is
- ✓ Ir-reflexive;
- ✓ Transitive.

(viii) A CCuqROFR $\bar{Ṛ}$ is called linear order, if it is
- ✓ Reflexive;
- ✓ Antisymmetric;
- ✓ Transitive;
- ✓ Complete.

(ix) Let $\bar{Ṛ}_1$ and $\bar{Ṛ}_2$ are two CCuqROFRs, then $\bar{Ṛ}_1 \circ \bar{Ṛ}_2$ is composite relation if,

$$\left\{(ţ,ḟ),\left(\begin{pmatrix}[\alpha^-_{(p)}(ţ,ḟ),\alpha^+_{(p)}(ţ,ḟ)]e^{[\beta^-_{(p)}(ţ,ḟ),\beta^+_{(p)}(ţ,ḟ)]лi},\\ [\alpha^-_{(n)}(ţ,ḟ),\alpha^+_{(n)}(ţ,ḟ)]e^{[\beta^-_{(n)}(ţ,ḟ),\beta^+_{(n)}(ţ,ḟ)]лi}\end{pmatrix},\begin{pmatrix}\alpha_{(p)}(ţ,ḟ)e^{\beta_{(p)}(ţ,ḟ)лi},\\ \alpha_{(n)}(ţ,ḟ)e^{\beta_{(n)}(ţ,ḟ)лi}\end{pmatrix}\right)\right\} \in \bar{Ṛ}_1, \text{ And,}$$

$$\left\{(ḟ,ḱ),\left(\begin{pmatrix}[\alpha^-_{(p)}(ḟ,ḱ),\alpha^+_{(p)}(ḟ,ḱ)]e^{[\beta^-_{(p)}(ḟ,ḱ),\beta^+_{(p)}(ḟ,ḱ)]лi},\\ [\alpha^-_{(n)}(ḟ,ḱ),\alpha^+_{(n)}(ḟ,ḱ)]e^{[\beta^-_{(n)}(ḟ,ḱ),\beta^+_{(n)}(ḟ,ḱ)]лi}\end{pmatrix},\begin{pmatrix}\alpha_{(p)}(ḟ,ḱ)e^{\beta_{(p)}(ḟ,ḱ)лi},\\ \alpha_{(n)}(ḟ,ḱ)e^{\beta_{(n)}(ḟ,ḱ)лi}\end{pmatrix}\right)\right\} \in \bar{Ṛ}_2, \text{ implies,}$$

$$(ţ,ḱ) = \left\{(ţ,ḱ),\left(\begin{pmatrix}[\alpha^-_{(p)}(ţ,ḱ),\alpha^+_{(p)}(ţ,ḱ)]e^{[\beta^-_{(p)}(ţ,ḱ),\beta^+_{(p)}(ţ,ḱ)]лi},\\ [\alpha^-_{(n)}(ţ,ḱ),\alpha^+_{(n)}(ţ,ḱ)]e^{[\beta^-_{(n)}(ţ,ḱ),\beta^+_{(n)}(ţ,ḱ)]лi}\end{pmatrix},\begin{pmatrix}\alpha_{(p)}(ţ,ḱ)e^{\beta_{(p)}(ţ,ḱ)лi},\\ \alpha_{(n)}(ţ,ḱ)e^{\beta_{(n)}(ţ,ḱ)лi}\end{pmatrix}\right)\right\}$$

Example 3: Let a CCuqROFS ϳ on Ù is defined as

$$ϳ = \left\{\begin{array}{c}\begin{pmatrix}ţ_1,\left([0.5,0.6]e^{[0.4,0.7]лi},[0.4,0.7]e^{[0.3,0.8]лi}\right),\\ \left(0.5e^{(0.5)лi},0.2e^{(0.5)лi}\right)\end{pmatrix},\\ \begin{pmatrix}ţ_2,\left([0.4,0.5]e^{[0.4,0.6]лi},[0.2,0.4]e^{[0.4,0.7]лi}\right),\\ \left(0.3e^{(0.4)лi},0.4e^{(0.6)лi}\right)\end{pmatrix},\\ \begin{pmatrix}ţ_3,\left([0.4,0.6]e^{[0.3,0.6]лi},[0.2,0.5]e^{[0.3,0.4]лi}\right),\\ \left(0.4e^{(0.5)лi},0.3e^{(0.5)лi}\right)\end{pmatrix}\end{array}\right\}$$

Then, its self-Cartesian product is

$$\text{j} \times \text{j} = \left\{ \begin{array}{c} \left( \begin{array}{c} (\text{ƫ}_1, \text{ƫ}_1), \left( [0.5,0.6] e^{[0.4,0.7]\text{л}i}, [0.4,0.7] e^{[0.3,0.8]\text{л}i} \right), \\ \left( 0.5 e^{(0.5)\text{л}i}, 0.2 e^{(0.5)\text{л}i} \right) \end{array} \right), \\ \left( \begin{array}{c} (\text{ƫ}_1, \text{ƫ}_2), \left( [0.4,0.5] e^{[0.4,0.6]\text{л}i}, [0.4,0.7] e^{[0.4,0.8]\text{л}i} \right), \\ \left( 0.3 e^{(0.4)\text{л}i}, 0.4 e^{(0.6)\text{л}i} \right) \end{array} \right), \\ \left( \begin{array}{c} (\text{ƫ}_1, \text{ƫ}_3), \left( [0.4,0.6] e^{[0.3,0.6]\text{л}i}, [0.4,0.7] e^{[0.3,0.8]\text{л}i} \right), \\ \left( 0.4 e^{(0.5)\text{л}i}, 0.3 e^{(0.5)\text{л}i} \right) \end{array} \right), \\ \left( \begin{array}{c} (\text{ƫ}_2, \text{ƫ}_1), \left( [0.4,0.5] e^{[0.4,0.6]\text{л}i}, [0.4,0.7] e^{[0.4,0.8]\text{л}i} \right), \\ \left( 0.3 e^{(0.4)\text{л}i}, 0.4 e^{(0.6)\text{л}i} \right) \end{array} \right), \\ \left( \begin{array}{c} (\text{ƫ}_2, \text{ƫ}_2), \left( [0.4,0.5] e^{[0.4,0.6]\text{л}i}, [0.2,0.4] e^{[0.4,0.7]\text{л}i} \right), \\ \left( 0.3 e^{(0.4)\text{л}i}, 0.4 e^{(0.6)\text{л}i} \right) \end{array} \right), \\ \left( \begin{array}{c} (\text{ƫ}_2, \text{ƫ}_3), \left( [0.4,0.5] e^{[0.3,0.6]\text{л}i}, [0.2,0.5] e^{[0.4,0.7]\text{л}i} \right), \\ \left( 0.3 e^{(0.4)\text{л}i}, 0.4 e^{(0.6)\text{л}i} \right) \end{array} \right), \\ \left( \begin{array}{c} (\text{ƫ}_3, \text{ƫ}_1), \left( [0.4,0.6] e^{[0.3,0.6]\text{л}i}, [0.4,0.7] e^{[0.3,0.8]\text{л}i} \right), \\ \left( 0.4 e^{(0.5)\text{л}i}, 0.3 e^{(0.5)\text{л}i} \right) \end{array} \right), \\ \left( \begin{array}{c} (\text{ƫ}_3, \text{ƫ}_2), \left( [0.4,0.5] e^{[0.3,0.6]\text{л}i}, [0.2,0.5] e^{[0.4,0.7]\text{л}i} \right), \\ \left( 0.3 e^{(0.4)\text{л}i}, 0.4 e^{(0.6)\text{л}i} \right) \end{array} \right), \\ \left( \begin{array}{c} (\text{ƫ}_3, \text{ƫ}_3), \left( [0.4,0.6] e^{[0.3,0.6]\text{л}i}, [0.2,0.5] e^{[0.3,0.4]\text{л}i} \right), \\ \left( 0.4 e^{(0.5)\text{л}i}, 0.3 e^{(0.5)\text{л}i} \right) \end{array} \right) \end{array} \right\}$$

(i) The reflexive relation $\bar{\text{Ṛ}}_1$ is

$$\bar{\text{Ṛ}}_1 = \left\{ \begin{array}{c} \left( \begin{array}{c} (\text{ƫ}_1, \text{ƫ}_1), \left( [0.5,0.6] e^{[0.4,0.7]\text{л}i}, [0.4,0.7] e^{[0.3,0.8]\text{л}i} \right), \\ \left( 0.5 e^{(0.5)\text{л}i}, 0.2 e^{(0.5)\text{л}i} \right) \end{array} \right), \\ \left( \begin{array}{c} (\text{ƫ}_2, \text{ƫ}_2), \left( [0.4,0.5] e^{[0.4,0.6]\text{л}i}, [0.2,0.4] e^{[0.4,0.7]\text{л}i} \right), \\ \left( 0.3 e^{(0.4)\text{л}i}, 0.4 e^{(0.6)\text{л}i} \right) \end{array} \right), \\ \left( \begin{array}{c} (\text{ƫ}_3, \text{ƫ}_3), \left( [0.4,0.6] e^{[0.3,0.6]\text{л}i}, [0.2,0.5] e^{[0.3,0.4]\text{л}i} \right), \\ \left( 0.4 e^{(0.5)\text{л}i}, 0.3 e^{(0.5)\text{л}i} \right) \end{array} \right) \end{array} \right\}$$

(ii) The symmetric relation $\bar{\text{Ṛ}}_2$ is

$$\bar{\text{Ṛ}}_2 = \left\{ \begin{array}{c} \left( \begin{array}{c} (\text{ƫ}_1, \text{ƫ}_1), \left( [0.5,0.6] e^{[0.4,0.7]\text{л}i}, [0.4,0.7] e^{[0.3,0.8]\text{л}i} \right), \\ \left( 0.5 e^{(0.5)\text{л}i}, 0.2 e^{(0.5)\text{л}i} \right) \end{array} \right), \\ \left( \begin{array}{c} (\text{ƫ}_1, \text{ƫ}_3), \left( [0.4,0.6] e^{[0.3,0.6]\text{л}i}, [0.4,0.7] e^{[0.3,0.8]\text{л}i} \right), \\ \left( 0.4 e^{(0.5)\text{л}i}, 0.3 e^{(0.5)\text{л}i} \right) \end{array} \right), \\ \left( \begin{array}{c} (\text{ƫ}_3, \text{ƫ}_1), \left( [0.4,0.6] e^{[0.3,0.6]\text{л}i}, [0.4,0.7] e^{[0.3,0.8]\text{л}i} \right), \\ \left( 0.4 e^{(0.5)\text{л}i}, 0.3 e^{(0.5)\text{л}i} \right) \end{array} \right), \\ \left( \begin{array}{c} (\text{ƫ}_2, \text{ƫ}_3), \left( [0.4,0.5] e^{[0.3,0.6]\text{л}i}, [0.2,0.5] e^{[0.4,0.7]\text{л}i} \right), \\ \left( 0.3 e^{(0.4)\text{л}i}, 0.4 e^{(0.6)\text{л}i} \right) \end{array} \right), \\ \left( \begin{array}{c} (\text{ƫ}_2, \text{ƫ}_2), \left( [0.4,0.5] e^{[0.4,0.6]\text{л}i}, [0.2,0.4] e^{[0.4,0.7]\text{л}i} \right), \\ \left( 0.3 e^{(0.4)\text{л}i}, 0.4 e^{(0.6)\text{л}i} \right) \end{array} \right), \\ \left( \begin{array}{c} (\text{ƫ}_3, \text{ƫ}_2), \left( [0.4,0.5] e^{[0.3,0.6]\text{л}i}, [0.2,0.5] e^{[0.4,0.7]\text{л}i} \right), \\ \left( 0.3 e^{(0.4)\text{л}i}, 0.4 e^{(0.6)\text{л}i} \right) \end{array} \right) \end{array} \right\}$$

(iii) The transitive relation $\bar{\text{Ṛ}}_3$ is

$$\underset{\cdot}{\bar{R}}_3=\left\{\begin{array}{c}\left(\begin{array}{c}(\text{ƫ}_1,\text{ƫ}_2),\left([0.4,0.5]e^{[0.4,0.6]\text{л}i},[0.4,0.7]e^{[0.4,0.8]\text{л}i}\right),\\ \left(0.3e^{(0.4)\text{л}i},0.4e^{(0.6)\text{л}i}\right)\end{array}\right),\\ \left(\begin{array}{c}(\text{ƫ}_2,\text{ƫ}_3),\left([0.4,0.5]e^{[0.3,0.6]\text{л}i},[0.2,0.5]e^{[0.4,0.7]\text{л}i}\right),\\ \left(0.3e^{(0.4)\text{л}i},0.4e^{(0.6)\text{л}i}\right)\end{array}\right),\\ \left(\begin{array}{c}(\text{ƫ}_1,\text{ƫ}_3),\left([0.4,0.6]e^{[0.3,0.6]\text{л}i},[0.4,0.7]e^{[0.3,0.8]\text{л}i}\right),\\ \left(0.4e^{(0.5)\text{л}i},0.3e^{(0.5)\text{л}i}\right)\end{array}\right)\end{array}\right\}$$

(iv) The equivalence relation $\underset{\cdot}{\bar{R}}_4$ is

$$\underset{\cdot}{\bar{R}}_4=\left\{\begin{array}{c}\left(\begin{array}{c}(\text{ƫ}_1,\text{ƫ}_1),\left([0.5,0.6]e^{[0.4,0.7]\text{л}i},[0.4,0.7]e^{[0.3,0.8]\text{л}i}\right),\\ \left(0.5e^{(0.5)\text{л}i},0.2e^{(0.5)\text{л}i}\right)\end{array}\right),\\ \left(\begin{array}{c}(\text{ƫ}_1,\text{ƫ}_2),\left([0.4,0.5]e^{[0.4,0.6]\text{л}i},[0.4,0.7]e^{[0.4,0.8]\text{л}i}\right),\\ \left(0.3e^{(0.4)\text{л}i},0.4e^{(0.6)\text{л}i}\right)\end{array}\right),\\ \left(\begin{array}{c}(\text{ƫ}_2,\text{ƫ}_1),\left([0.4,0.5]e^{[0.4,0.6]\text{л}i},[0.4,0.7]e^{[0.4,0.8]\text{л}i}\right),\\ \left(0.3e^{(0.4)\text{л}i},0.4e^{(0.6)\text{л}i}\right)\end{array}\right),\\ \left(\begin{array}{c}(\text{ƫ}_2,\text{ƫ}_2),\left([0.4,0.5]e^{[0.4,0.6]\text{л}i},[0.2,0.4]e^{[0.4,0.7]\text{л}i}\right),\\ \left(0.3e^{(0.4)\text{л}i},0.4e^{(0.6)\text{л}i}\right)\end{array}\right),\\ \left(\begin{array}{c}(\text{ƫ}_3,\text{ƫ}_3),\left([0.4,0.6]e^{[0.3,0.6]\text{л}i},[0.2,0.5]e^{[0.3,0.4]\text{л}i}\right),\\ \left(0.4e^{(0.5)\text{л}i},0.3e^{(0.5)\text{л}i}\right)\end{array}\right)\end{array}\right\}$$

(v) The partial order relation $\underset{\cdot}{\bar{R}}_5$ is

$$\underset{\cdot}{\bar{R}}_5=\left\{\begin{array}{c}\left(\begin{array}{c}(\text{ƫ}_1,\text{ƫ}_1),\left([0.5,0.6]e^{[0.4,0.7]\text{л}i},[0.4,0.7]e^{[0.3,0.8]\text{л}i}\right),\\ \left(0.5e^{(0.5)\text{л}i},0.2e^{(0.5)\text{л}i}\right)\end{array}\right),\\ \left(\begin{array}{c}(\text{ƫ}_1,\text{ƫ}_2),\left([0.4,0.5]e^{[0.4,0.6]\text{л}i},[0.4,0.7]e^{[0.4,0.8]\text{л}i}\right),\\ \left(0.3e^{(0.4)\text{л}i},0.4e^{(0.6)\text{л}i}\right)\end{array}\right),\\ \left(\begin{array}{c}(\text{ƫ}_1,\text{ƫ}_3),\left([0.4,0.6]e^{[0.3,0.6]\text{л}i},[0.4,0.7]e^{[0.3,0.8]\text{л}i}\right),\\ \left(0.4e^{(0.5)\text{л}i},0.3e^{(0.5)\text{л}i}\right)\end{array}\right),\\ \left(\begin{array}{c}(\text{ƫ}_2,\text{ƫ}_2),\left([0.4,0.5]e^{[0.4,0.6]\text{л}i},[0.2,0.4]e^{[0.4,0.7]\text{л}i}\right),\\ \left(0.3e^{(0.4)\text{л}i},0.4e^{(0.6)\text{л}i}\right)\end{array}\right),\\ \left(\begin{array}{c}(\text{ƫ}_3,\text{ƫ}_2),\left([0.4,0.5]e^{[0.3,0.6]\text{л}i},[0.2,0.5]e^{[0.4,0.7]\text{л}i}\right),\\ \left(0.3e^{(0.4)\text{л}i},0.4e^{(0.6)\text{л}i}\right)\end{array}\right),\\ \left(\begin{array}{c}(\text{ƫ}_3,\text{ƫ}_3),\left([0.4,0.6]e^{[0.3,0.6]\text{л}i},[0.2,0.5]e^{[0.3,0.4]\text{л}i}\right),\\ \left(0.4e^{(0.5)\text{л}i},0.3e^{(0.5)\text{л}i}\right)\end{array}\right)\end{array}\right\}$$

(vi) Let $\underset{\cdot}{\bar{R}}_a$ and $\underset{\cdot}{\bar{R}}_b$ be two CCuqROFSs,

$$\underset{\cdot}{\bar{R}}_a=\left\{\begin{array}{c}\left(\begin{array}{c}(\text{ƫ}_1,\text{ƫ}_2),\left([0.4,0.5]e^{[0.4,0.6]\text{л}i},[0.4,0.7]e^{[0.4,0.8]\text{л}i}\right),\\ \left(0.3e^{(0.4)\text{л}i},0.4e^{(0.6)\text{л}i}\right)\end{array}\right),\\ \left(\begin{array}{c}(\text{ƫ}_2,\text{ƫ}_1),\left([0.4,0.5]e^{[0.4,0.6]\text{л}i},[0.4,0.7]e^{[0.4,0.8]\text{л}i}\right),\\ \left(0.3e^{(0.4)\text{л}i},0.4e^{(0.6)\text{л}i}\right)\end{array}\right),\\ \left(\begin{array}{c}(\text{ƫ}_3,\text{ƫ}_2),\left([0.4,0.5]e^{[0.3,0.6]\text{л}i},[0.2,0.5]e^{[0.4,0.7]\text{л}i}\right),\\ \left(0.3e^{(0.4)\text{л}i},0.4e^{(0.6)\text{л}i}\right)\end{array}\right)\end{array}\right\}$$

$$\bar{Ṛ}_b = \left\{ \begin{array}{c} \left( \begin{array}{c} (ƫ_1, ƫ_3), \left([0.4,0.6]e^{[0.3,0.6]\pi i}, [0.4,0.7]e^{[0.3,0.8]\pi i}\right), \\ \left(0.4e^{(0.5)\pi i}, 0.3e^{(0.5)\pi i}\right) \end{array} \right), \\ \left( \begin{array}{c} (ƫ_2, ƫ_1), \left([0.4,0.5]e^{[0.4,0.6]\pi i}, [0.4,0.7]e^{[0.4,0.8]\pi i}\right), \\ \left(0.3e^{(0.4)\pi i}, 0.4e^{(0.6)\pi i}\right) \end{array} \right), \\ \left( \begin{array}{c} (ƫ_3, ƫ_2), \left([0.4,0.5]e^{[0.3,0.6]\pi i}, [0.2,0.5]e^{[0.4,0.7]\pi i}\right), \\ \left(0.3e^{(0.4)\pi i}, 0.4e^{(0.6)\pi i}\right) \end{array} \right), \\ \left( \begin{array}{c} (ƫ_3, ƫ_3), \left([0.4,0.6]e^{[0.3,0.6]\pi i}, [0.2,0.5]e^{[0.3,0.4]\pi i}\right), \\ \left(0.4e^{(0.5)\pi i}, 0.3e^{(0.5)\pi i}\right) \end{array} \right) \end{array} \right\}$$

Then, the CCuqRO composite fuzzy relations $\bar{Ṛ}_a o \bar{Ṛ}_b$ is defined as,

$$\bar{Ṛ}_a o \bar{Ṛ}_b = \left\{ \begin{array}{c} \left( \begin{array}{c} (ƫ_1, ƫ_1), \left([0.5,0.6]e^{[0.4,0.7]\pi i}, [0.4,0.7]e^{[0.3,0.8]\pi i}\right), \\ \left(0.5e^{(0.5)\pi i}, 0.2e^{(0.5)\pi i}\right) \end{array} \right), \\ \left( \begin{array}{c} (ƫ_2, ƫ_3), \left([0.4,0.5]e^{[0.3,0.6]\pi i}, [0.2,0.5]e^{[0.4,0.7]\pi i}\right), \\ \left(0.3e^{(0.4)\pi i}, 0.4e^{(0.6)\pi i}\right) \end{array} \right), \\ \left( \begin{array}{c} (ƫ_3, ƫ_1), \left([0.4,0.6]e^{[0.3,0.6]\pi i}, [0.4,0.7]e^{[0.3,0.8]\pi i}\right), \\ \left(0.4e^{(0.5)\pi i}, 0.3e^{(0.5)\pi i}\right) \end{array} \right) \end{array} \right\}$$

Theorem 1. A CCuqROFR $\bar{Ṛ}$ is a CCuqRO symmetric fuzzy relation on a CCuqROFS Ƙ if and only if $\bar{Ṛ} = \bar{Ṛ}^{-1}$.

*Proof*: Suppose that $\bar{Ṛ}$ is a CCuqRO symmetric fuzzy relation on a CCuqROFS Ƙ, then

$$\forall \left\{ (ƫ, ḟ), \left( \left( \begin{array}{c} \left[\alpha^-_{(p)}(ƫ, ḟ), \alpha^+_{(p)}(ƫ, ḟ)\right] e^{\left[\beta^-_{(p)}(ƫ,ḟ), \beta^+_{(p)}(ƫ,ḟ)\right]\pi i}, \\ \left[\alpha^-_{(n)}(ƫ, ḟ), \alpha^+_{(n)}(ƫ, ḟ)\right] e^{\left[\beta^-_{(n)}(ƫ,ḟ), \beta^+_{(n)}(ƫ,ḟ)\right]\pi i} \end{array} \right), \left( \begin{array}{c} \alpha_{(p)}(ƫ, ḟ) e^{\beta_{(p)}(ƫ,ḟ)\pi i}, \\ \alpha_{(n)}(ƫ, ḟ) e^{\beta_{(n)}(ƫ,ḟ)\pi i} \end{array} \right) \right) \right\} \in \bar{Ṛ},$$

$$\text{And} \left\{ (ḟ, ƫ), \left( \left( \begin{array}{c} \left[\alpha^-_{(p)}(ḟ, ƫ), \alpha^+_{(p)}(ḟ, ƫ)\right] e^{\left[\beta^-_{(p)}(ḟ,ƫ), \beta^+_{(p)}(ḟ,ƫ)\right]\pi i}, \\ \left[\alpha^-_{(n)}(ḟ, ƫ), \alpha^+_{(n)}(ḟ, ƫ)\right] e^{\left[\beta^-_{(n)}(ḟ,ƫ), \beta^+_{(n)}(ḟ,ƫ)\right]\pi i} \end{array} \right), \left( \begin{array}{c} \alpha_{(p)}(ḟ, ƫ) e^{\beta_{(p)}(ḟ,ƫ)\pi i}, \\ \alpha_{(n)}(ḟ, ƫ) e^{\beta_{(n)}(ḟ,ƫ)\pi i} \end{array} \right) \right) \right\} \in \bar{Ṛ}.$$

But

$$\left\{ (ḟ, ƫ), \left( \left( \begin{array}{c} \left[\alpha^-_{(p)}(ḟ, ƫ), \alpha^+_{(p)}(ḟ, ƫ)\right] e^{\left[\beta^-_{(p)}(ḟ,ƫ), \beta^+_{(p)}(ḟ,ƫ)\right]\pi i}, \\ \left[\alpha^-_{(n)}(ḟ, ƫ), \alpha^+_{(n)}(ḟ, ƫ)\right] e^{\left[\beta^-_{(n)}(ḟ,ƫ), \beta^+_{(n)}(ḟ,ƫ)\right]\pi i} \end{array} \right), \left( \begin{array}{c} \alpha_{(p)}(ḟ, ƫ) e^{\beta_{(p)}(ḟ,ƫ)\pi i}, \\ \alpha_{(n)}(ḟ, ƫ) e^{\beta_{(n)}(ḟ,ƫ)\pi i} \end{array} \right) \right) \right\} \in \bar{Ṛ}^{-1}.$$

Hence, $\bar{Ṛ} = \bar{Ṛ}^{-1}$

Conversely, suppose that , $\bar{Ṛ} = \bar{Ṛ}^{-1}$, then

$$\forall \left\{ (ƫ, ḟ), \left( \left( \begin{array}{c} \left[\alpha^-_{(p)}(ƫ, ḟ), \alpha^+_{(p)}(ƫ, ḟ)\right] e^{\left[\beta^-_{(p)}(ƫ,ḟ), \beta^+_{(p)}(ƫ,ḟ)\right]\pi i}, \\ \left[\alpha^-_{(n)}(ƫ, ḟ), \alpha^+_{(n)}(ƫ, ḟ)\right] e^{\left[\beta^-_{(n)}(ƫ,ḟ), \beta^+_{(n)}(ƫ,ḟ)\right]\pi i} \end{array} \right), \left( \begin{array}{c} \alpha_{(p)}(ƫ, ḟ) e^{\beta_{(p)}(ƫ,ḟ)\pi i}, \\ \alpha_{(n)}(ƫ, ḟ) e^{\beta_{(n)}(ƫ,ḟ)\pi i} \end{array} \right) \right) \right\} \in \bar{Ṛ}, \text{ we have}$$

$$\left\{ (ḟ, ƫ), \left( \left( \begin{array}{c} \left[\alpha^-_{(p)}(ḟ, ƫ), \alpha^+_{(p)}(ḟ, ƫ)\right] e^{\left[\beta^-_{(p)}(ḟ,ƫ), \beta^+_{(p)}(ḟ,ƫ)\right]\pi i}, \\ \left[\alpha^-_{(n)}(ḟ, ƫ), \alpha^+_{(n)}(ḟ, ƫ)\right] e^{\left[\beta^-_{(n)}(ḟ,ƫ), \beta^+_{(n)}(ḟ,ƫ)\right]\pi i} \end{array} \right), \left( \begin{array}{c} \alpha_{(p)}(ḟ, ƫ) e^{\beta_{(p)}(ḟ,ƫ)\pi i}, \\ \alpha_{(n)}(ḟ, ƫ) e^{\beta_{(n)}(ḟ,ƫ)\pi i} \end{array} \right) \right) \right\} \in \bar{Ṛ}^{-1}.$$

$$\left\{ (ḟ, ƫ), \left( \left( \begin{array}{c} \left[\alpha^-_{(p)}(ḟ, ƫ), \alpha^+_{(p)}(ḟ, ƫ)\right] e^{\left[\beta^-_{(p)}(ḟ,ƫ), \beta^+_{(p)}(ḟ,ƫ)\right]\pi i}, \\ \left[\alpha^-_{(n)}(ḟ, ƫ), \alpha^+_{(n)}(ḟ, ƫ)\right] e^{\left[\beta^-_{(n)}(ḟ,ƫ), \beta^+_{(n)}(ḟ,ƫ)\right]\pi i} \end{array} \right), \left( \begin{array}{c} \alpha_{(p)}(ḟ, ƫ) e^{\beta_{(p)}(ḟ,ƫ)\pi i}, \\ \alpha_{(n)}(ḟ, ƫ) e^{\beta_{(n)}(ḟ,ƫ)\pi i} \end{array} \right) \right) \right\} \in \bar{Ṛ}$$

So, $\bar{Ṛ}$ is a CCuqRO symmetric fuzzy relation on Ƙ.

Theorem 2. A CCuqROFR $\bar{Ṛ}$ is a CCuqRO transitive fuzzy relation on a CCuqROFS Ƙ if and only if $\bar{Ṛ} \; o \; \bar{Ṛ} \subseteq \bar{Ṛ}$.

*Proof*: Suppose that Ṝ is a CCuqRO transitive fuzzy relation on a CCuqROFS Ҡ, then

$$\left\{(ţ,ḱ),\left(\begin{pmatrix}[\alpha^-_{(p)}(ţ,ḱ),\alpha^+_{(p)}(ţ,ḱ)]e^{[\beta^-_{(p)}(ţ,ḱ),\beta^+_{(p)}(ţ,ḱ)]лi},\\ [\alpha^-_{(n)}(ţ,ḱ),\alpha^+_{(n)}(ţ,ḱ)]e^{[\beta^-_{(n)}(ţ,ḱ),\beta^+_{(n)}(ţ,ḱ)]лi}\end{pmatrix},\begin{pmatrix}\alpha_{(p)}(ţ,ḱ)e^{\beta_{(p)}(ţ,ḱ)лi},\\ \alpha_{(n)}(ţ,ḱ)e^{\beta_{(n)}(ţ,ḱ)лi}\end{pmatrix}\right)\right\}\in Ṝ\ o\ Ṝ$$

There exists an element ḟ ∈ Ṝ, then by the transitivity of Ṝ

$$\left\{(ţ,ḟ),\left(\begin{pmatrix}[\alpha^-_{(p)}(ţ,ḟ),\alpha^+_{(p)}(ţ,ḟ)]e^{[\beta^-_{(p)}(ţ,ḟ),\beta^+_{(p)}(ţ,ḟ)]лi},\\ [\alpha^-_{(n)}(ţ,ḟ),\alpha^+_{(n)}(ţ,ḟ)]e^{[\beta^-_{(n)}(ţ,ḟ),\beta^+_{(n)}(ţ,ḟ)]лi}\end{pmatrix},\begin{pmatrix}\alpha_{(p)}(ţ,ḟ)e^{\beta_{(p)}(ţ,ḟ)лi},\\ \alpha_{(n)}(ţ,ḟ)e^{\beta_{(n)}(ţ,ḟ)лi}\end{pmatrix}\right)\right\}\in Ṝ, \text{ and}$$

$$\left\{(ḟ,ḱ),\left(\begin{pmatrix}[\alpha^-_{(p)}(ḟ,ḱ),\alpha^+_{(p)}(ḟ,ḱ)]e^{[\beta^-_{(p)}(ḟ,ḱ),\beta^+_{(p)}(ḟ,ḱ)]лi},\\ [\alpha^-_{(n)}(ḟ,ḱ),\alpha^+_{(n)}(ḟ,ḱ)]e^{[\beta^-_{(n)}(ḟ,ḱ),\beta^+_{(n)}(ḟ,ḱ)]лi}\end{pmatrix},\begin{pmatrix}\alpha_{(p)}(ḟ,ḱ)e^{\beta_{(p)}(ḟ,ḱ)лi},\\ \alpha_{(n)}(ḟ,ḱ)e^{\beta_{(n)}(ḟ,ḱ)лi}\end{pmatrix}\right)\right\}\in Ṝ, \text{ implies,}$$

$$\left\{(ţ,ḱ),\left(\begin{pmatrix}[\alpha^-_{(p)}(ţ,ḱ),\alpha^+_{(p)}(ţ,ḱ)]e^{[\beta^-_{(p)}(ţ,ḱ),\beta^+_{(p)}(ţ,ḱ)]лi},\\ [\alpha^-_{(n)}(ţ,ḱ),\alpha^+_{(n)}(ţ,ḱ)]e^{[\beta^-_{(n)}(ţ,ḱ),\beta^+_{(n)}(ţ,ḱ)]лi}\end{pmatrix},\begin{pmatrix}\alpha_{(p)}(ţ,ḱ)e^{\beta_{(p)}(ţ,ḱ)лi},\\ \alpha_{(n)}(ţ,ḱ)e^{\beta_{(n)}(ţ,ḱ)лi}\end{pmatrix}\right)\right\}\in Ṝ$$

Hence, Ṝ *o* Ṝ ⊆ Ṝ.

Conversely, let us assume that Ṝ *o* Ṝ ⊆ Ṝ, then the composition of CCuqROFR implies that,

$$\left\{(ţ,ḟ),\left(\begin{pmatrix}[\alpha^-_{(p)}(ţ,ḟ),\alpha^+_{(p)}(ţ,ḟ)]e^{[\beta^-_{(p)}(ţ,ḟ),\beta^+_{(p)}(ţ,ḟ)]лi},\\ [\alpha^-_{(n)}(ţ,ḟ),\alpha^+_{(n)}(ţ,ḟ)]e^{[\beta^-_{(n)}(ţ,ḟ),\beta^+_{(n)}(ţ,ḟ)]лi}\end{pmatrix},\begin{pmatrix}\alpha_{(p)}(ţ,ḟ)e^{\beta_{(p)}(ţ,ḟ)лi},\\ \alpha_{(n)}(ţ,ḟ)e^{\beta_{(n)}(ţ,ḟ)лi}\end{pmatrix}\right)\right\}\in Ṝ, \text{ and}$$

$$\left\{(ḟ,ḱ),\left(\begin{pmatrix}[\alpha^-_{(p)}(ḟ,ḱ),\alpha^+_{(p)}(ḟ,ḱ)]e^{[\beta^-_{(p)}(ḟ,ḱ),\beta^+_{(p)}(ḟ,ḱ)]лi},\\ [\alpha^-_{(n)}(ḟ,ḱ),\alpha^+_{(n)}(ḟ,ḱ)]e^{[\beta^-_{(n)}(ḟ,ḱ),\beta^+_{(n)}(ḟ,ḱ)]лi}\end{pmatrix},\begin{pmatrix}\alpha_{(p)}(ḟ,ḱ)e^{\beta_{(p)}(ḟ,ḱ)лi},\\ \alpha_{(n)}(ḟ,ḱ)e^{\beta_{(n)}(ḟ,ḱ)лi}\end{pmatrix}\right)\right\}\in Ṝ, \text{ then,}$$

$$\left\{(ţ,ḱ),\left(\begin{pmatrix}[\alpha^-_{(p)}(ţ,ḱ),\alpha^+_{(p)}(ţ,ḱ)]e^{[\beta^-_{(p)}(ţ,ḱ),\beta^+_{(p)}(ţ,ḱ)]лi},\\ [\alpha^-_{(n)}(ţ,ḱ),\alpha^+_{(n)}(ţ,ḱ)]e^{[\beta^-_{(n)}(ţ,ḱ),\beta^+_{(n)}(ţ,ḱ)]лi}\end{pmatrix},\begin{pmatrix}\alpha_{(p)}(ţ,ḱ)e^{\beta_{(p)}(ţ,ḱ)лi},\\ \alpha_{(n)}(ţ,ḱ)e^{\beta_{(n)}(ţ,ḱ)лi}\end{pmatrix}\right)\right\}\in Ṝ$$

Theorem 3. If Ṝ is a CCuqRO equivalence fuzzy relation on a CCuqROFS Ҡ, then Ṝ *o* Ṝ = Ṝ.

*Proof*: As Ṝ is a CCuqRO equivalence fuzzy relation on a CCuqROFS Ҡ,

$$\text{For }\left\{(ţ,ḟ),\left(\begin{pmatrix}[\alpha^-_{(p)}(ţ,ḟ),\alpha^+_{(p)}(ţ,ḟ)]e^{[\beta^-_{(p)}(ţ,ḟ),\beta^+_{(p)}(ţ,ḟ)]лi},\\ [\alpha^-_{(n)}(ţ,ḟ),\alpha^+_{(n)}(ţ,ḟ)]e^{[\beta^-_{(n)}(ţ,ḟ),\beta^+_{(n)}(ţ,ḟ)]лi}\end{pmatrix},\begin{pmatrix}\alpha_{(p)}(ţ,ḟ)e^{\beta_{(p)}(ţ,ḟ)лi},\\ \alpha_{(n)}(ţ,ḟ)e^{\beta_{(n)}(ţ,ḟ)лi}\end{pmatrix}\right)\right\}\in Ṝ.$$

The CCuqRO symmetric fuzzy relation implies that,

$$\left\{(ḟ,ţ),\left(\begin{pmatrix}[\alpha^-_{(p)}(ḟ,ţ),\alpha^+_{(p)}(ḟ,ţ)]e^{[\beta^-_{(p)}(ḟ,ţ),\beta^+_{(p)}(ḟ,ţ)]лi},\\ [\alpha^-_{(n)}(ḟ,ţ),\alpha^+_{(n)}(ḟ,ţ)]e^{[\beta^-_{(n)}(ḟ,ţ),\beta^+_{(n)}(ḟ,ţ)]лi}\end{pmatrix},\begin{pmatrix}\alpha_{(p)}(ḟ,ţ)e^{\beta_{(p)}(ḟ,ţ)лi},\\ \alpha_{(n)}(ḟ,ţ)e^{\beta_{(n)}(ḟ,ţ)лi}\end{pmatrix}\right)\right\}\in Ṝ.$$

The CCuqRO transitive fuzzy relation implies that,

$$\left\{(ţ,ţ),\left(\begin{pmatrix}[\alpha^-_{(p)}(ţ,ţ),\alpha^+_{(p)}(ţ,ţ)]e^{[\beta^-_{(p)}(ţ,ţ),\beta^+_{(p)}(ţ,ţ)]лi},\\ [\alpha^-_{(n)}(ţ,ţ),\alpha^+_{(n)}(ţ,ţ)]e^{[\beta^-_{(n)}(ţ,ţ),\beta^+_{(n)}(ţ,ţ)]лi}\end{pmatrix},\begin{pmatrix}\alpha_{(p)}(ţ,ţ)e^{\beta_{(p)}(ţ,ţ)лi},\\ \alpha_{(n)}(ţ,ţ)e^{\beta_{(n)}(ţ,ţ)лi}\end{pmatrix}\right)\right\}\in Ṝ.$$

Also, CCuqRO composite fuzzy relation implies that

$$\left\{(ţ,ţ),\left(\begin{pmatrix}[\alpha^-_{(p)}(ţ,ţ),\alpha^+_{(p)}(ţ,ţ)]e^{[\beta^-_{(p)}(ţ,ţ),\beta^+_{(p)}(ţ,ţ)]лi},\\ [\alpha^-_{(n)}(ţ,ţ),\alpha^+_{(n)}(ţ,ţ)]e^{[\beta^-_{(n)}(ţ,ţ),\beta^+_{(n)}(ţ,ţ)]лi}\end{pmatrix},\begin{pmatrix}\alpha_{(p)}(ţ,ţ)e^{\beta_{(p)}(ţ,ţ)лi},\\ \alpha_{(n)}(ţ,ţ)e^{\beta_{(n)}(ţ,ţ)лi}\end{pmatrix}\right)\right\}\in Ṝ\ o\ Ṝ$$

Therefore, Ṝ ⊆ Ṝ *o* Ṝ (1).

Conversely, suppose that,

$$\left\{(\text{ţ},\acute{k}),\left(\begin{pmatrix}[\alpha^-_{(p)}(\text{ţ},\acute{k}),\alpha^+_{(p)}(\text{ţ},\acute{k})]e^{[\beta^-_{(p)}(\text{ţ},\acute{k}),\beta^+_{(p)}(\text{ţ},\acute{k})]\text{л}i},\\ [\alpha^-_{(n)}(\text{ţ},\acute{k}),\alpha^+_{(n)}(\text{ţ},\acute{k})]e^{[\beta^-_{(n)}(\text{ţ},\acute{k}),\beta^+_{(n)}(\text{ţ},\acute{k})]\text{л}i}\end{pmatrix},\begin{pmatrix}\alpha_{(p)}(\text{ţ},\acute{k})e^{\beta_{(p)}(\text{ţ},\acute{k})\text{л}i},\\ \alpha_{(n)}(\text{ţ},\acute{k})e^{\beta_{(n)}(\text{ţ},\acute{k})\text{л}i}\end{pmatrix}\right)\right\}\in \bar{\underset{\cdot}{R}}\ o\ \bar{\underset{\cdot}{R}}$$

Then there exist an element $\dot{f}$ in

$$\left\{(\text{ţ},\dot{f}),\left(\begin{pmatrix}[\alpha^-_{(p)}(\text{ţ},\dot{f}),\alpha^+_{(p)}(\text{ţ},\dot{f})]e^{[\beta^-_{(p)}(\text{ţ},\dot{f}),\beta^+_{(p)}(\text{ţ},\dot{f})]\text{л}i},\\ [\alpha^-_{(n)}(\text{ţ},\dot{f}),\alpha^+_{(n)}(\text{ţ},\dot{f})]e^{[\beta^-_{(n)}(\text{ţ},\dot{f}),\beta^+_{(n)}(\text{ţ},\dot{f})]\text{л}i}\end{pmatrix},\begin{pmatrix}\alpha_{(p)}(\text{ţ},\dot{f})e^{\beta_{(p)}(\text{ţ},\dot{f})\text{л}i},\\ \alpha_{(n)}(\text{ţ},\dot{f})e^{\beta_{(n)}(\text{ţ},\dot{f})\text{л}i}\end{pmatrix}\right)\right\}\in \bar{\underset{\cdot}{R}}.$$

$$\left\{(\dot{f},\text{ţ}),\left(\begin{pmatrix}[\alpha^-_{(p)}(\dot{f},\text{ţ}),\alpha^+_{(p)}(\dot{f},\text{ţ})]e^{[\beta^-_{(p)}(\dot{f},\text{ţ}),\beta^+_{(p)}(\dot{f},\text{ţ})]\text{л}i},\\ [\alpha^-_{(n)}(\dot{f},\text{ţ}),\alpha^+_{(n)}(\dot{f},\text{ţ})]e^{[\beta^-_{(n)}(\dot{f},\text{ţ}),\beta^+_{(n)}(\dot{f},\text{ţ})]\text{л}i}\end{pmatrix},\begin{pmatrix}\alpha_{(p)}(\dot{f},\text{ţ})e^{\beta_{(p)}(\dot{f},\text{ţ})\text{л}i},\\ \alpha_{(n)}(\dot{f},\text{ţ})e^{\beta_{(n)}(\dot{f},\text{ţ})\text{л}i}\end{pmatrix}\right)\right\}\in \bar{\underset{\cdot}{R}}.$$

Since $\bar{\underset{\cdot}{R}}$ is a CCuqRO equivalence fuzzy relation. Hence, the CCuqRO transitive fuzzy relation $\bar{\underset{\cdot}{R}}$ implies that,

$$\left\{(\text{ţ},\acute{k}),\left(\begin{pmatrix}[\alpha^-_{(p)}(\text{ţ},\acute{k}),\alpha^+_{(p)}(\text{ţ},\acute{k})]e^{[\beta^-_{(p)}(\text{ţ},\acute{k}),\beta^+_{(p)}(\text{ţ},\acute{k})]\text{л}i},\\ [\alpha^-_{(n)}(\text{ţ},\acute{k}),\alpha^+_{(n)}(\text{ţ},\acute{k})]e^{[\beta^-_{(n)}(\text{ţ},\acute{k}),\beta^+_{(n)}(\text{ţ},\acute{k})]\text{л}i}\end{pmatrix},\begin{pmatrix}\alpha_{(p)}(\text{ţ},\acute{k})e^{\beta_{(p)}(\text{ţ},\acute{k})\text{л}i},\\ \alpha_{(n)}(\text{ţ},\acute{k})e^{\beta_{(n)}(\text{ţ},\acute{k})\text{л}i}\end{pmatrix}\right)\right\}\in \bar{\underset{\cdot}{R}}.$$

Therefore, $\bar{\underset{\cdot}{R}}\ o\ \bar{\underset{\cdot}{R}} \subseteq \bar{\underset{\cdot}{R}}$. (2). Hence proved $\bar{\underset{\cdot}{R}}\ o\ \bar{\underset{\cdot}{R}} = \bar{\underset{\cdot}{R}}$.

## 4. Application

In this section, the application of the proposed concepts is discussed. The CCuqROFS, CCuqROFR, and their types are used in this study of web services applications. We observe cybersecurity and cyber threats on web services.

### *4.1 Web Services*

A software component known as a web service enables extensible, networked machine-to-machine connectivity. It has a comprehensive robot architecture. Web services are used to fulfil a certain task or set of tasks. The work of online services permeates every aspect of our daily lives. They are used for a variety of actions that are already second nature to us, like purchasing goods and services online and logging onto social media accounts.

Some examples of web services are as below:

*i) Web Services Metadata Exchange*

A web services protocol standard called WS-Metadata Exchange was released by BEA Systems, IBM, Microsoft, and SAP. The WS-Meta Data Transfer standard is intended to function in concert with WS-Addressing, WSDL, and WS-Policy to enable the retrieval of metadata about a Web Services endpoint. It is a component of the WS-Federation agenda. Metadata can be classified as either descriptive, administrative, or structural. Resource discovery, classification, and selection are made possible by descriptive information.

*ii) Web Services Description Language*

Networked, XML-based services are described using the Web Services Descriptive Language (WSDL), a standard specification. It allows service providers a simple mechanism to communicate the essential structure of requests to their systems, irrespective of their fundamental run-time implementations. An XML-based protocol for information exchange in distributed and decentralised systems is called WSDL. The operations that a web service will carry out are described in WSDL specifications, along with how to access it. A language called WSDL is used to specify how to connect to XML-based services.

*iii) Web Services Conversation Language*

The public operations or corporate level talks that a Web service supports can be defined using the Web Services Conversation Language (WSCL). The XML documents that are being exchanged, as well as the permitted order in which they must be shared, are specified by WSCL. Because WSCL conversation definitions are XML documents in and of themselves, Web Services architectures and application frameworks can understand them.

*iv) Web Services Flow Language*

The flow of messages in Web services is described using the domain-specific language known as Web Services Flow Language (WSFL). It is an XML-based language that is used to specify the order in which a Web service performs its actions as well as the circumstances in which they are carried out. The goal of WSFL is to give developers a high-level, abstract representation of how messages go through a Web service, freeing them from having to worry about the implementation's finer points.

*4.2 Securities of the Web Services*

A wide range of security requirements, such as authentication, permission, privacy, trust, truthfulness, and confidentiality, as well as secure communication networks, federation, representatives, and auditing across a variety of application and business topologies, must be taken into account when securing web services. Some of the securities of the web services are discussed below:

*i) Ensuring Transport Confidentiality (ETC)*

Confidentiality guarantees that an eavesdropper has not intercepted the information flow. Integrity refers to the absence of unauthorised intervention in the content during the transmission of information from the sender to the recipient. In addition to potentially affecting your ability to land a job in the future, maintaining confidentiality is important for ethical, moral, and legal reasons. In many nations, certain information, such as "trade secrets" and personally identifiable information, is protected by law.

$$\left(\text{ETC}, \left([0.4,0.6]e^{[0.3,0.6]\pi i}, [0.4,0.7]e^{[0.3,0.8]\pi i}\right), \left(0.3e^{(0.4)\pi i}, 0.4e^{(0.6)\pi i}\right)\right)$$

*ii) Maintaining Message Integrity (MMI)*

When a transmission is deemed to be comprehensible, it hasn't been tampered with or changed. Using a hash function, which integrates all the message's bytes with a secret key to produce a message digest that is difficult to reverse, is the most often used technique. Checking the message's legitimacy is what the term "Message Integrity" refers to. It checks to see if the message hasn't been tampered with or altered. A communication is said to be intelligible if it has not been tampered with or altered. The reliability of a message can be checked in a variety of ways.

$$\left(\text{MMI}, \left([0.6,0.7]e^{[0.5,0.9]\pi i}, [0.5,0.7]e^{[0.4,0.7]\pi i}\right), \left(0.4e^{(0.6)\pi i}, 0.6e^{(0.8)\pi i}\right)\right)$$

*iii) Authentication Best Practice (ABP)*

Emphasizing passwordless identification and implementing unified login or SSO with a two-factor authentication are the best recommendations for securing the verification process. To build safeguards that stop logical defects, security considerations must be made early in the design process. The most widely used authentication method that goes "beyond passwords" is the implementation of multi-factor authentication (MFA), often known as two-factor identification or two-step verification (2FA).

$$\left(\text{ABP}, \left([0.5,0.8]e^{[0.4,0.7]\pi i}, [0.2,0.7]e^{[0.8,0.9]\pi i}\right), \left(0.5e^{(0.7)\pi i}, 0.7e^{(0.8)\pi i}\right)\right)$$

*iv) Transport Encoding (TE)*

Transfer-Encoding is a hop-by-hop header that is used to encrypt data in messages sent between two nodes rather than the actual resource. A multi-node connection's segments can each utilize a distinct Transfer-Encoding value. Use the end-to-end Content-Encoding header if you wish to compress data across the entire connection. Encoding is used to change data so that it can be correctly (and safely) viewed by a different kind of system, such as seeing special characters on a web page or sending binary data via email. The objective is to make sure that knowledge can be appropriately ingested, not to keep it a secret.

$$\left(\text{TE}, \left([0.4,0.9]e^{[0.2,0.8]\pi i}, [0.3,0.6]e^{[0.7,0.9]\pi i}\right), \left(0.1e^{(0.9)\pi i}, 0.6e^{(0.9)\pi i}\right)\right)$$

*v) Message Confidentiality (MC)*

Message confidentiality refers to the fact that only the sender and the recipient are aware of the contents of a message. This is distinct from message integrity, which denotes that both the sender and recipient of a message are who they claim to be and that the message has not been tampered with. The idea of confidentiality is to hide or encrypt your data so that only the intended receiver may access it. This is typically accomplished using encryption. Data that has not yet been encrypted is referred to as plain text, sometimes known as clear text. Once the data has been encrypted, it is referred to as Cipher text.

$$\left(\text{MC}, \left([0.3,0.6]e^{[0.3,0.7]\pi i}, [0.4,0.7]e^{[0.5,0.7]\pi i}\right), \left(0.3e^{(0.8)\pi i}, 0.5e^{(0.7)\pi i}\right)\right)$$

*vi) Schema Validation (SV)*

A request's target endpoint and HTTP method are just two of the request properties that an API schema uses to define which API requests are legitimate. You can use schema validation to determine whether incoming traffic corresponds with an API schema that was previously provided. A unique, object-based method of defining request validations or sanitizations is through schemas. Field routes are specified as keys and objects are specified as values at the root level, defining the warning messages, placements, and confirmations and sanitizations.

$$\left(\text{SV}, \left([0.2,0.3]e^{[0.3,0.5]\pi i}, [0.5,0.6]e^{[0.5,0.8]\pi i}\right), \left(0.7e^{(0.8)\pi i}, 0.6e^{(0.7)\pi i}\right)\right)$$

Therefore, the CCuqROFS j summarizing the security is given as follows:

$$\text{j} = \left\{\begin{array}{c} \left(ETC, \left([0.4,0.6]e^{[0.3,0.6]\pi i}, [0.4,0.7]e^{[0.3,0.8]\pi i}\right), \left(0.3e^{(0.4)\pi i}, 0.4e^{(0.6)\pi i}\right)\right), \\ \left(\text{MMI}, \left([0.6,0.7]e^{[0.5,0.9]\pi i}, [0.5,0.7]e^{[0.4,0.7]\pi i}\right), \left(0.4e^{(0.6)\pi i}, 0.6e^{(0.8)\pi i}\right)\right), \\ \left(\text{ABP}, \left([0.5,0.8]e^{[0.4,0.7]\pi i}, [0.2,0.7]e^{[0.8,0.9]\pi i}\right), \left(0.5e^{(0.7)\pi i}, 0.7e^{(0.8)\pi i}\right)\right), \\ \left(\text{TE}, \left([0.4,0.9]e^{[0.2,0.8]\pi i}, [0.3,0.6]e^{[0.7,0.9]\pi i}\right), \left(0.1e^{(0.9)\pi i}, 0.6e^{(0.9)\pi i}\right)\right), \\ \left(\text{MC}, \left([0.3,0.6]e^{[0.3,0.7]\pi i}, [0.4,0.7]e^{[0.5,0.7]\pi i}\right), \left(0.3e^{(0.8)\pi i}, 0.5e^{(0.7)\pi i}\right)\right), \\ \left(\text{SV}, \left([0.2,0.3]e^{[0.3,0.5]\pi i}, [0.5,0.6]e^{[0.5,0.8]\pi i}\right), \left(0.7e^{(0.8)\pi i}, 0.6e^{(0.7)\pi i}\right)\right) \end{array}\right\}$$

*4.3 Threats of the Web Services*

Where media manipulation is the human side of web dangers, harmful code is the technological side. These dangers may consist of, but not be limited to: Injection attacks: Introduction of dangerous scripts into legitimate programmes and websites. Following are the most common types of web services threats:

*i) Viruses and Worms (VW)*

Until their host file is triggered, viruses are latent. Once inside the system, worms can multiply themselves and spread on their own. On computers or across computer networks, viruses and caterpillars can replicate themselves even without user's knowledge, and every new instance of these dangerous programmes has the capability to do the same.

$$\left(\mathrm{VW}, \left([0.7,0.8]e^{[0.3,0.4]лi}, [0.4,0.8]e^{[0.6,0.9]лi}\right), \left(0.7e^{(0.9)лi}, 0.5e^{(0.8)лi}\right)\right)$$

*ii) Ransomware (RW)*

Ransomware is a type of harmful malware that locks down computers and stops users from using them until a ransom is paid. As they have been around for a while, ransomware variants typically attempt to extort money from its victims by displaying an on-screen alert. Ransomware is a type of malware that uses access to its victims' computers' data to extract money from them. The two most prevalent types of cyberattacks are encryptors and screen locks.

$$\left(\mathrm{RW}, \left([0.5,0.7]e^{[0.4,0.5]лi}, [0.5,0.7]e^{[0.7,0.9]лi}\right), \left(0.6e^{(0.8)лi}, 0.4e^{(0.7)лi}\right)\right)$$

*iii) CEO Fraud and Impersonation (CFI)*

CEO fraud is a sort of cybercrime in which criminals pose as corporate executives in an effort to deceive a worker into submitting illicit wire transfers or disclosing confidential information. A spear phishing email assault known as CEO Fraud involves the attacker pretending to be your CEO. The attacker typically tries to mislead you into sending money to a bank account they own, sending them critical HR information, or disclosing other confidential material.

$$\left(\mathrm{CFI}, \left([0.3,0.7]e^{[0.6,0.8]лi}, [0.5,0.8]e^{[0.6,0.9]лi}\right), \left(0.4e^{(0.7)лi}, 0.5e^{(0.8)лi}\right)\right)$$

*iv) Cryptographic Failures (CF)*

A cryptographic failure, a major vulnerability in web application security, exposes private application data to a subpar or non-existent cryptographic system. Examples include passwords, patient medical information, trade secrets, credit card numbers, email addresses, and other private user data. When you do not adequately protect sensitive data, such as usernames, credit card numbers, and personal information, attackers frequently target these shortcomings. This is the main reason why sensitive data has been exposed.

$$\left(\mathrm{CF}, \left([0.4,0.8]e^{[0.5,0.7]лi}, [0.4,0.6]e^{[0.5,0.7]лi}\right), \left(0.3e^{(0.6)лi}, 0.4e^{(0.7)лi}\right)\right)$$

*v) Broken Access Control (BAC)*

An unauthorized person can access restricted resources thanks to a security issue known as broken access control vulnerabilities. Attackers can get around regular security measures and obtain unauthorised access to sensitive data or systems by taking advantage of this vulnerability.

$$\left(\mathrm{BAC}, \left([0.3,0.7]e^{[0.6,0.7]лi}, [0.5,0.6]e^{[0.5,0.8]лi}\right), \left(0.5e^{(0.6)лi}, 0.6e^{(0.7)лi}\right)\right)$$

*vi) Cross-Site Scripting (CSS)*

Cross site scripting (CSS) is a type of attack where a perpetrator inserts harmful executable scripts into the source code of a reliable website or application. Attackers frequently start a CSS attack by providing a user a dangerous link and tempting them to click it.

$$\left(\text{CSS}, \left([0.3,0.5]e^{[0.5,0.6]лi}, [0.2,0.8]e^{[0.7,0.9]лi}\right), \left(0.6e^{(0.7)лi}, 0.8e^{(0.9)лi}\right)\right)$$

Therefore, the CCuqROFS $\mathbb{H}$ summarizing the threat is given as follows:

$$\mathbb{H} = \left\{\begin{array}{c} \left(VW, \left([0.7,0.8]e^{[0.3,0.4]лi}, [0.4,0.8]e^{[0.6,0.9]лi}\right), \left(0.7e^{(0.9)лi}, 0.5e^{(0.8)лi}\right)\right), \\ \left(\text{RW}, \left([0.5,0.7]e^{[0.4,0.5]лi}, [0.5,0.7]e^{[0.7,0.9]лi}\right), \left(0.6e^{(0.8)лi}, 0.4e^{(0.7)лi}\right)\right), \\ \left(\text{CFI}, \left([0.3,0.7]e^{[0.6,0.8]лi}, [0.5,0.8]e^{[0.6,0.9]лi}\right), \left(0.4e^{(0.7)лi}, 0.5e^{(0.8)лi}\right)\right), \\ \left(\text{CF}, \left([0.4,0.8]e^{[0.5,0.7]лi}, [0.4,0.6]e^{[0.5,0.7]лi}\right), \left(0.3e^{(0.6)лi}, 0.4e^{(0.7)лi}\right)\right), \\ \left(\text{BAC}, \left([0.3,0.7]e^{[0.6,0.7]лi}, [0.5,0.6]e^{[0.5,0.8]лi}\right), \left(0.5e^{(0.6)лi}, 0.6e^{(0.7)лi}\right)\right), \\ \left(\text{CSS}, \left([0.3,0.5]e^{[0.5,0.6]лi}, [0.2,0.8]e^{[0.7,0.9]лi}\right), \left(0.6e^{(0.7)лi}, 0.8e^{(0.9)лi}\right)\right) \end{array}\right\}$$

We use the following mathematics to examine the effectiveness and ineffectiveness of every web services security measure and threat. We have the following two CCuqROFSs $j$ and $\mathbb{H}$, corresponding to the set of security and threats, respectively.

$$\text{j} = \left\{\begin{array}{c} \left(ETC, \left([0.4,0.6]e^{[0.3,0.6]лi}, [0.4,0.7]e^{[0.3,0.8]лi}\right), \left(0.3e^{(0.4)лi}, 0.4e^{(0.6)лi}\right)\right), \\ \left(\text{MMI}, \left([0.6,0.7]e^{[0.5,0.9]лi}, [0.5,0.7]e^{[0.4,0.7]лi}\right), \left(0.4e^{(0.6)лi}, 0.6e^{(0.8)лi}\right)\right), \\ \left(\text{ABP}, \left([0.5,0.8]e^{[0.4,0.7]лi}, [0.2,0.7]e^{[0.8,0.9]лi}\right), \left(0.5e^{(0.7)лi}, 0.7e^{(0.8)лi}\right)\right), \\ \left(\text{TE}, \left([0.4,0.9]e^{[0.2,0.8]лi}, [0.3,0.6]e^{[0.7,0.9]лi}\right), \left(0.1e^{(0.9)лi}, 0.6e^{(0.9)лi}\right)\right), \\ \left(\text{MC}, \left([0.3,0.6]e^{[0.3,0.7]лi}, [0.4,0.7]e^{[0.5,0.7]лi}\right), \left(0.3e^{(0.8)лi}, 0.5e^{(0.7)лi}\right)\right), \\ \left(\text{SV}, \left([0.2,0.3]e^{[0.3,0.5]лi}, [0.5,0.6]e^{[0.5,0.8]лi}\right), \left(0.7e^{(0.8)лi}, 0.6e^{(0.7)лi}\right)\right) \end{array}\right\}$$

$$\mathbb{H} = \left\{\begin{array}{c} \left(VW, \left([0.7,0.8]e^{[0.3,0.4]лi}, [0.4,0.8]e^{[0.6,0.9]лi}\right), \left(0.7e^{(0.9)лi}, 0.5e^{(0.8)лi}\right)\right), \\ \left(\text{RW}, \left([0.5,0.7]e^{[0.4,0.5]лi}, [0.5,0.7]e^{[0.7,0.9]лi}\right), \left(0.6e^{(0.8)лi}, 0.4e^{(0.7)лi}\right)\right), \\ \left(\text{CFI}, \left([0.3,0.7]e^{[0.6,0.8]лi}, [0.5,0.8]e^{[0.6,0.9]лi}\right), \left(0.4e^{(0.7)лi}, 0.5e^{(0.8)лi}\right)\right), \\ \left(\text{CF}, \left([0.4,0.8]e^{[0.5,0.7]лi}, [0.4,0.6]e^{[0.5,0.7]лi}\right), \left(0.3e^{(0.6)лi}, 0.4e^{(0.7)лi}\right)\right), \\ \left(\text{BAC}, \left([0.3,0.7]e^{[0.6,0.7]лi}, [0.5,0.6]e^{[0.5,0.8]лi}\right), \left(0.5e^{(0.6)лi}, 0.6e^{(0.7)лi}\right)\right), \\ \left(\text{CSS}, \left([0.3,0.5]e^{[0.5,0.6]лi}, [0.2,0.8]e^{[0.7,0.9]лi}\right), \left(0.6e^{(0.7)лi}, 0.8e^{(0.9)лi}\right)\right) \end{array}\right\}$$

Then the Cartesian product of securities and threats are as follows,

$$
\mathrm{j} \times \mathbb{H} = \left\{ \begin{array}{c}
\left((ETC, VW), \left([0.4,0.6]e^{[0.3,0.4]\pi i}, [0.4,0.8]e^{[0.6,0.9]\pi i}\right), \left(0.3e^{(0.4)\pi i}, 0.5e^{(0.8)\pi i}\right)\right), \\
\left((ETC, RW), \left([0.4,0.6]e^{[0.3,0.5]\pi i}, [0.5,0.7]e^{[0.7,0.9]\pi i}\right), \left(0.3e^{(0.4)\pi i}, 0.4e^{(0.7)\pi i}\right)\right), \\
\left((ETC, CFI), \left([0.3,0.6]e^{[0.3,0.6]\pi i}, [0.5,0.8]e^{[0.6,0.9]\pi i}\right), \left(0.3e^{(0.4)\pi i}, 0.5e^{(0.8)\pi i}\right)\right), \\
\left((ETC, CF), \left([0.3,0.6]e^{[0.3,0.6]\pi i}, [0.4,0.7]e^{[0.5,0.8]\pi i}\right), \left(0.3e^{(0.4)\pi i}, 0.4e^{(0.7)\pi i}\right)\right), \\
\left((ETC, BAC), \left([0.3,0.6]e^{[0.3,0.6]\pi i}, [0.5,0.7]e^{[0.5,0.8]\pi i}\right), \left(0.3e^{(0.4)\pi i}, 0.6e^{(0.7)\pi i}\right)\right), \\
\left((ETC, BAC), \left([0.3,0.5]e^{[0.3,0.6]\pi i}, [0.4,0.8]e^{[0.7,0.9]\pi i}\right), \left(0.3e^{(0.4)\pi i}, 0.8e^{(0.9)\pi i}\right)\right), \\
\left((MMI, VW), \left([0.6,0.7]e^{[0.3,0.4]\pi i}, [0.5,0.8]e^{[0.6,0.9]\pi i}\right), \left(0.4e^{(0.6)\pi i}, 0.6e^{(0.8)\pi i}\right)\right), \\
\left((MMI, RW), \left([0.5,0.7]e^{[0.4,0.5]\pi i}, [0.5,0.7]e^{[0.7,0.9]\pi i}\right), \left(0.4e^{(0.6)\pi i}, 0.6e^{(0.8)\pi i}\right)\right), \\
\left((MMI, CFI), \left([0.3,0.7]e^{[0.6,0.8]\pi i}, [0.5,0.8]e^{[0.6,0.9]\pi i}\right), \left(0.4e^{(0.6)\pi i}, 0.6e^{(0.8)\pi i}\right)\right), \\
\left((MMI, CF), \left([0.4,0.7]e^{[0.5,0.7]\pi i}, [0.5,0.7]e^{[0.5,0.7]\pi i}\right), \left(0.3e^{(0.6)\pi i}, 0.6e^{(0.8)\pi i}\right)\right), \\
\left((MMI, BAC), \left([0.3,0.7]e^{[0.5,0.7]\pi i}, [0.5,0.8]e^{[0.5,0.8]\pi i}\right), \left(0.4e^{(0.6)\pi i}, 0.6e^{(0.8)\pi i}\right)\right), \\
\left((MMI, CSS), \left([0.3,0.5]e^{[0.5,0.6]\pi i}, [0.5,0.8]e^{[0.7,0.9]\pi i}\right), \left(0.4e^{(0.6)\pi i}, 0.8e^{(0.9)\pi i}\right)\right), \\
\left((ABP, VW), \left([0.5,0.8]e^{[0.3,0.4]\pi i}, [0.4,0.8]e^{[0.8,0.9]\pi i}\right), \left(0.5e^{(0.7)\pi i}, 0.7e^{(0.8)\pi i}\right)\right), \\
\left((ABP, RW), \left([0.5,0.7]e^{[0.4,0.5]\pi i}, [0.5,0.7]e^{[0.8,0.9]\pi i}\right), \left(0.5e^{(0.7)\pi i}, 0.7e^{(0.8)\pi i}\right)\right), \\
\left((ABP, CFI), \left([0.3,0.7]e^{[0.4,0.7]\pi i}, [0.5,0.8]e^{[0.8,0.9]\pi i}\right), \left(0.4e^{(0.7)\pi i}, 0.7e^{(0.8)\pi i}\right)\right), \\
\left((ABP, CF), \left([0.4,0.8]e^{[0.4,0.7]\pi i}, [0.4,0.7]e^{[0.8,0.9]\pi i}\right), \left(0.3e^{(0.6)\pi i}, 0.7e^{(0.8)\pi i}\right)\right), \\
\left((ABP, BAC), \left([0.5,0.7]e^{[0.4,0.5]\pi i}, [0.5,0.7]e^{[0.8,0.9]\pi i}\right), \left(0.5e^{(0.7)\pi i}, 0.7e^{(0.8)\pi i}\right)\right), \\
\left((ABP, CSS), \left([0.3,0.5]e^{[0.4,0.6]\pi i}, [0.2,0.8]e^{[0.8,0.9]\pi i}\right), \left(0.5e^{(0.7)\pi i}, 0.8e^{(0.9)\pi i}\right)\right), \\
\left((TE, VW), \left([0.4,0.8]e^{[0.2,0.4]\pi i}, [0.4,0.8]e^{[0.7,0.9]\pi i}\right), \left(0.1e^{(0.9)\pi i}, 0.6e^{(0.9)\pi i}\right)\right), \\
\left((TE, RW), \left([0.4,0.7]e^{[0.2,0.5]\pi i}, [0.5,0.7]e^{[0.7,0.9]\pi i}\right), \left(0.1e^{(0.8)\pi i}, 0.6e^{(0.9)\pi i}\right)\right), \\
\left((TE, CFI), \left([0.3,0.7]e^{[0.2,0.8]\pi i}, [0.5,0.8]e^{[0.7,0.9]\pi i}\right), \left(0.1e^{(0.7)\pi i}, 0.6e^{(0.9)\pi i}\right)\right), \\
\left((TE, CF), \left([0.4,0.8]e^{[0.2,0.7]\pi i}, [0.4,0.6]e^{[0.7,0.9]\pi i}\right), \left(0.1e^{(0.6)\pi i}, 0.6e^{(0.9)\pi i}\right)\right), \\
\left((TE, BAC), \left([0.3,0.7]e^{[0.2,0.7]\pi i}, [0.5,0.6]e^{[0.7,0.9]\pi i}\right), \left(0.1e^{(0.6)\pi i}, 0.6e^{(0.9)\pi i}\right)\right), \\
\left((TE, CSS), \left([0.4,0.8]e^{[0.2,0.4]\pi i}, [0.4,0.8]e^{[0.7,0.9]\pi i}\right), \left(0.1e^{(0.9)\pi i}, 0.6e^{(0.9)\pi i}\right)\right), \\
\left((MC, VW), \left([0.3,0.6]e^{[0.3,0.4]\pi i}, [0.4,0.8]e^{[0.6,0.9]\pi i}\right), \left(0.3e^{(0.8)\pi i}, 0.5e^{(0.8)\pi i}\right)\right), \\
\left((MC, RW), \left([0.3,0.6]e^{[0.3,0.5]\pi i}, [0.5,0.7]e^{[0.7,0.9]\pi i}\right), \left(0.3e^{(0.8)\pi i}, 0.5e^{(0.7)\pi i}\right)\right), \\
\left((MC, CFI), \left([0.3,0.6]e^{[0.3,0.7]\pi i}, [0.5,0.8]e^{[0.6,0.9]\pi i}\right), \left(0.3e^{(0.7)\pi i}, 0.5e^{(0.8)\pi i}\right)\right), \\
\left((MC, CF), \left([0.3,0.6]e^{[0.3,0.7]\pi i}, [0.4,0.7]e^{[0.5,0.7]\pi i}\right), \left(0.3e^{(0.6)\pi i}, 0.5e^{(0.7)\pi i}\right)\right), \\
\left((MC, BAC), \left([0.3,0.6]e^{[0.3,0.7]\pi i}, [0.5,0.7]e^{[0.5,0.8]\pi i}\right), \left(0.3e^{(0.6)\pi i}, 0.6e^{(0.7)\pi i}\right)\right), \\
\left((MC, CSS), \left([0.3,0.5]e^{[0.3,0.6]\pi i}, [0.4,0.8]e^{[0.7,0.9]\pi i}\right), \left(0.3e^{(0.7)\pi i}, 0.8e^{(0.9)\pi i}\right)\right), \\
\left((SV, VW), \left([0.2,0.3]e^{[0.3,0.4]\pi i}, [0.5,0.8]e^{[0.6,0.9]\pi i}\right), \left(0.7e^{(0.8)\pi i}, 0.6e^{(0.8)\pi i}\right)\right) \\
\left((SV, RW), \left([0.2,0.3]e^{[0.3,0.5]\pi i}, [0.5,0.7]e^{[0.7,0.9]\pi i}\right), \left(0.6e^{(0.8)\pi i}, 0.6e^{(0.7)\pi i}\right)\right) \\
\left((SV, CFI), \left([0.2,0.3]e^{[0.3,0.5]\pi i}, [0.5,0.8]e^{[0.6,0.9]\pi i}\right), \left(0.4e^{(0.7)\pi i}, 0.6e^{(0.8)\pi i}\right)\right) \\
\left((SV, CF), \left([0.2,0.3]e^{[0.3,0.5]\pi i}, [0.4,0.6]e^{[0.6,0.7]\pi i}\right), \left(0.3e^{(0.6)\pi i}, 0.6e^{(0.7)\pi i}\right)\right) \\
\left((SV, BAC), \left([0.2,0.3]e^{[0.3,0.5]\pi i}, [0.5,0.6]e^{[0.5,0.8]\pi i}\right), \left(0.5e^{(0.6)\pi i}, 0.6e^{(0.7)\pi i}\right)\right) \\
\left((SV, CSS), \left([0.2,0.3]e^{[0.3,0.5]\pi i}, [0.5,0.8]e^{[0.7,0.9]\pi i}\right), \left(0.6e^{(0.7)\pi i}, 0.8e^{(0.9)\pi i}\right)\right)
\end{array} \right\}
$$

We assign levels of M, NM, and interval-valued levels of M, NM, and threats to each web service. These functions in sets $j$ and $\mathbb{H}$ depict the effects of both threats and securities in the present as well as the future. The efficacy of each security precaution against a specific threat is determined by the Cartesian product between the CCuqROFS $j$ and $\mathbb{H}$. The above calculations shows the Cartesian product between the CCuqROFS $j$ and $\mathbb{H}$, The relationship among each element in a set, i.e., the state and effect of a security on a threat, is described by the Cartesian product $j \times \mathbb{H}$. The levels of a M show how efficient a web services safety system is in identifying a specific threat over a duration of time. The degree of NM levels indicates an ineffectiveness of security against a specific threat over time. For example, the ordered pair,

$$\left((ABP, BAC), \left([0.5,0.7]e^{[0.4,0.5]\text{л}i}, [0.5,0.7]e^{[0.3,0.4]\text{л}i}\right), \left(0.5e^{(0.7)\text{л}i}, 0.7e^{(0.8)\text{л}i}\right)\right)$$

describes the authentication best practice against broken access control. Because of this, the $ABP$ safeguards the $BAC$ by preventing unwanted access to the user's device. It also discusses how the ordered pair will affect the present and the future. Because security effectiveness outperforms degree of ineffectiveness, the $ABP$ minimises the threat of the $BAC$ in the present. A certain ordered pair makes an interval-based prediction about the future security.

Example 4. Let j be the set of securities and $\mathbb{H}$ be the set of threats,

$$j = \left\{\begin{array}{l}
ţ_1, \left(\begin{array}{c}\left([0.4,0.6]e^{[0.3,0.6]\text{л}i}, [0.4,0.7]e^{[0.3,0.8]\text{л}i}\right),\\ \left(0.3e^{(0.4)\text{л}i}, 0.4e^{(0.6)\text{л}i}\right)\end{array}\right), \left(\begin{array}{c}\left([0.5,0.7]e^{[0.4,0.7]\text{л}i}, [0.5,0.8]e^{[0.4,0.9]\text{л}i}\right),\\ \left(0.6e^{(0.7)\text{л}i}, 0.6e^{(0.8)\text{л}i}\right)\end{array}\right), \left(\begin{array}{c}\left([0.3,0.7]e^{[0.5,0.7]\text{л}i}, [0.6,0.8]e^{[0.4,0.8]\text{л}i}\right),\\ \left(0.5e^{(0.9)\text{л}i}, 0.6e^{(0.7)\text{л}i}\right)\end{array}\right), \left(\begin{array}{c}\left([0.4,0.7]e^{[0.6,0.7]\text{л}i}, [0.6,0.8]e^{[0.4,0.8]\text{л}i}\right),\\ \left(0.5e^{(0.9)\text{л}i}, 0.7e^{(0.8)\text{л}i}\right)\end{array}\right)\\
ţ_2, \left(\begin{array}{c}\left([0.6,0.7]e^{[0.5,0.9]\text{л}i}, [0.5,0.7]e^{[0.4,0.7]\text{л}i}\right),\\ \left(0.4e^{(0.6)\text{л}i}, 0.6e^{(0.8)\text{л}i}\right)\end{array}\right), \left(\begin{array}{c}\left([0.5,0.7]e^{[0.4,0.5]\text{л}i}, [0.5,0.7]e^{[0.7,0.9]\text{л}i}\right),\\ \left(0.6e^{(0.8)\text{л}i}, 0.4e^{(0.7)\text{л}i}\right)\end{array}\right), \left(\begin{array}{c}\left([0.7,0.9]e^{[0.5,0.7]\text{л}i}, [0.5,0.9]e^{[0.6,0.9]\text{л}i}\right),\\ \left(0.5e^{(0.7)\text{л}i}, 0.8e^{(0.9)\text{л}i}\right)\end{array}\right), \left(\begin{array}{c}\left([0.7,0.8]e^{[0.3,0.4]\text{л}i}, [0.4,0.8]e^{[0.6,0.9]\text{л}i}\right),\\ \left(0.7e^{(0.9)\text{л}i}, 0.5e^{(0.8)\text{л}i}\right)\end{array}\right)\\
ţ_3, \left(\begin{array}{c}\left([0.5,0.8]e^{[0.4,0.7]\text{л}i}, [0.2,0.7]e^{[0.8,0.9]\text{л}i}\right),\\ \left(0.5e^{(0.7)\text{л}i}, 0.7e^{(0.8)\text{л}i}\right)\end{array}\right), \left(\begin{array}{c}\left([0.7,0.8]e^{[0.3,0.4]\text{л}i}, [0.4,0.8]e^{[0.6,0.9]\text{л}i}\right),\\ \left(0.7e^{(0.9)\text{л}i}, 0.5e^{(0.8)\text{л}i}\right)\end{array}\right), \left(\begin{array}{c}\left([0.5,0.7]e^{[0.4,0.5]\text{л}i}, [0.5,0.7]e^{[0.7,0.9]\text{л}i}\right),\\ \left(0.6e^{(0.8)\text{л}i}, 0.4e^{(0.7)\text{л}i}\right)\end{array}\right), \left(\begin{array}{c}\left([0.5,0.7]e^{[0.4,0.7]\text{л}i}, [0.5,0.8]e^{[0.4,0.9]\text{л}i}\right),\\ \left(0.6e^{(0.7)\text{л}i}, 0.6e^{(0.8)\text{л}i}\right)\end{array}\right)\\
ţ_4, \left(\begin{array}{c}\left([0.4,0.9]e^{[0.2,0.8]\text{л}i}, [0.3,0.6]e^{[0.7,0.9]\text{л}i}\right),\\ \left(0.1e^{(0.9)\text{л}i}, 0.6e^{(0.9)\text{л}i}\right)\end{array}\right), \left(\begin{array}{c}\left([0.2,0.3]e^{[0.3,0.5]\text{л}i}, [0.5,0.6]e^{[0.5,0.8]\text{л}i}\right),\\ \left(0.7e^{(0.8)\text{л}i}, 0.6e^{(0.7)\text{л}i}\right)\end{array}\right), \left(\begin{array}{c}\left([0.5,0.7]e^{[0.4,0.7]\text{л}i}, [0.5,0.8]e^{[0.4,0.9]\text{л}i}\right),\\ \left(0.6e^{(0.7)\text{л}i}, 0.6e^{(0.8)\text{л}i}\right)\end{array}\right), \left(\begin{array}{c}\left([0.5,0.8]e^{[0.4,0.7]\text{л}i}, [0.2,0.7]e^{[0.8,0.9]\text{л}i}\right),\\ \left(0.5e^{(0.7)\text{л}i}, 0.7e^{(0.8)\text{л}i}\right)\end{array}\right)\\
ţ_5, \left(\begin{array}{c}\left([0.3,0.6]e^{[0.3,0.7]\text{л}i}, [0.4,0.7]e^{[0.5,0.7]\text{л}i}\right),\\ \left(0.3e^{(0.8)\text{л}i}, 0.5e^{(0.7)\text{л}i}\right)\end{array}\right), \left(\begin{array}{c}\left([0.2,0.7]e^{[0.6,0.8]\text{л}i}, [0.5,0.9]e^{[0.6,0.9]\text{л}i}\right),\\ \left(0.4e^{(0.7)\text{л}i}, 0.3e^{(0.6)\text{л}i}\right)\end{array}\right), \left(\begin{array}{c}\left([0.4,0.8]e^{[0.4,0.8]\text{л}i}, [0.6,0.7]e^{[0.5,0.9]\text{л}i}\right),\\ \left(0.6e^{(0.8)\text{л}i}, 0.5e^{(0.9)\text{л}i}\right)\end{array}\right), \left(\begin{array}{c}\left([0.2,0.3]e^{[0.3,0.5]\text{л}i}, [0.5,0.6]e^{[0.5,0.8]\text{л}i}\right),\\ \left(0.7e^{(0.8)\text{л}i}, 0.6e^{(0.7)\text{л}i}\right)\end{array}\right)\\
ţ_6, \left(\begin{array}{c}\left([0.2,0.3]e^{[0.3,0.5]\text{л}i}, [0.5,0.6]e^{[0.5,0.8]\text{л}i}\right),\\ \left(0.7e^{(0.8)\text{л}i}, 0.6e^{(0.7)\text{л}i}\right)\end{array}\right), \left(\begin{array}{c}\left([0.3,0.7]e^{[0.6,0.7]\text{л}i}, [0.5,0.6]e^{[0.5,0.8]\text{л}i}\right),\\ \left(0.5e^{(0.6)\text{л}i}, 0.6e^{(0.7)\text{л}i}\right)\end{array}\right), \left(\begin{array}{c}\left([0.6,0.7]e^{[0.5,0.9]\text{л}i}, [0.5,0.7]e^{[0.4,0.7]\text{л}i}\right),\\ \left(0.4e^{(0.6)\text{л}i}, 0.6e^{(0.8)\text{л}i}\right)\end{array}\right), \left(\begin{array}{c}\left([0.7,0.9]e^{[0.5,0.7]\text{л}i}, [0.5,0.9]e^{[0.6,0.9]\text{л}i}\right),\\ \left(0.5e^{(0.7)\text{л}i}, 0.8e^{(0.9)\text{л}i}\right)\end{array}\right)
\end{array}\right\}$$

$$\mathbb{H} = \left\{ \begin{array}{l}
ţ_1, \left(\begin{pmatrix}[0.7,0.8]e^{[0.3,0.4]\text{л}i}, \\ [0.4,0.8]e^{[0.6,0.9]\text{л}i}\end{pmatrix}, \begin{pmatrix}0.7e^{(0.9)\text{л}i}, \\ 0.5e^{(0.8)\text{л}i}\end{pmatrix}\right), \left(\begin{pmatrix}[0.6,0.7]e^{[0.2,0.4]\text{л}i}, \\ [0.3,0.9]e^{[0.4,0.7]\text{л}i}\end{pmatrix}, \begin{pmatrix}0.5e^{(0.9)\text{л}i}, \\ 0.4e^{(0.8)\text{л}i}\end{pmatrix}\right), \left(\begin{pmatrix}[0.4,0.8]e^{[0.4,0.8]\text{л}i}, \\ [0.6,0.7]e^{[0.5,0.9]\text{л}i}\end{pmatrix}, \begin{pmatrix}0.6e^{(0.8)\text{л}i}, \\ 0.5e^{(0.9)\text{л}i}\end{pmatrix}\right), \left(\begin{pmatrix}[0.7,0.8]e^{[0.3,0.4]\text{л}i}, \\ [0.4,0.8]e^{[0.6,0.9]\text{л}i}\end{pmatrix}, \begin{pmatrix}0.7e^{(0.9)\text{л}i}, \\ 0.5e^{(0.8)\text{л}i}\end{pmatrix}\right) \\
ţ_2, \left(\begin{pmatrix}[0.5,0.7]e^{[0.4,0.5]\text{л}i}, \\ [0.5,0.7]e^{[0.7,0.9]\text{л}i}\end{pmatrix}, \begin{pmatrix}0.6e^{(0.8)\text{л}i}, \\ 0.4e^{(0.7)\text{л}i}\end{pmatrix}\right), \left(\begin{pmatrix}[0.5,0.7]e^{[0.4,0.7]\text{л}i}, \\ [0.5,0.8]e^{[0.4,0.9]\text{л}i}\end{pmatrix}, \begin{pmatrix}0.6e^{(0.7)\text{л}i}, \\ 0.6e^{(0.8)\text{л}i}\end{pmatrix}\right), \left(\begin{pmatrix}[0.4,0.9]e^{[0.2,0.8]\text{л}i}, \\ [0.3,0.6]e^{[0.7,0.9]\text{л}i}\end{pmatrix}, \begin{pmatrix}0.1e^{(0.9)\text{л}i}, \\ 0.6e^{(0.9)\text{л}i}\end{pmatrix}\right), \left(\begin{pmatrix}[0.7,0.8]e^{[0.3,0.4]\text{л}i}, \\ [0.4,0.8]e^{[0.6,0.9]\text{л}i}\end{pmatrix}, \begin{pmatrix}0.7e^{(0.9)\text{л}i}, \\ 0.5e^{(0.8)\text{л}i}\end{pmatrix}\right) \\
ţ_3, \left(\begin{pmatrix}[0.3,0.7]e^{[0.6,0.8]\text{л}i}, \\ [0.5,0.8]e^{[0.6,0.9]\text{л}i}\end{pmatrix}, \begin{pmatrix}0.4e^{(0.7)\text{л}i}, \\ 0.5e^{(0.8)\text{л}i}\end{pmatrix}\right), \left(\begin{pmatrix}[0.2,0.7]e^{[0.6,0.8]\text{л}i}, \\ [0.3,0.8]e^{[0.6,0.9]\text{л}i}\end{pmatrix}, \begin{pmatrix}0.4e^{(0.5)\text{л}i}, \\ 0.5e^{(0.9)\text{л}i}\end{pmatrix}\right), \left(\begin{pmatrix}[0.7,0.8]e^{[0.3,0.4]\text{л}i}, \\ [0.4,0.8]e^{[0.6,0.9]\text{л}i}\end{pmatrix}, \begin{pmatrix}0.7e^{(0.9)\text{л}i}, \\ 0.5e^{(0.8)\text{л}i}\end{pmatrix}\right), \left(\begin{pmatrix}[0.2,0.7]e^{[0.6,0.8]\text{л}i}, \\ [0.5,0.9]e^{[0.6,0.9]\text{л}i}\end{pmatrix}, \begin{pmatrix}0.4e^{(0.7)\text{л}i}, \\ 0.3e^{(0.6)\text{л}i}\end{pmatrix}\right) \\
ţ_4, \left(\begin{pmatrix}[0.4,0.8]e^{[0.5,0.7]\text{л}i}, \\ [0.4,0.6]e^{[0.5,0.7]\text{л}i}\end{pmatrix}, \begin{pmatrix}0.3e^{(0.6)\text{л}i}, \\ 0.4e^{(0.7)\text{л}i}\end{pmatrix}\right), \left(\begin{pmatrix}[0.2,0.7]e^{[0.6,0.8]\text{л}i}, \\ [0.5,0.9]e^{[0.6,0.9]\text{л}i}\end{pmatrix}, \begin{pmatrix}0.4e^{(0.7)\text{л}i}, \\ 0.3e^{(0.6)\text{л}i}\end{pmatrix}\right), \left(\begin{pmatrix}[0.4,0.9]e^{[0.2,0.8]\text{л}i}, \\ [0.3,0.6]e^{[0.7,0.9]\text{л}i}\end{pmatrix}, \begin{pmatrix}0.1e^{(0.9)\text{л}i}, \\ 0.6e^{(0.9)\text{л}i}\end{pmatrix}\right), \left(\begin{pmatrix}[0.4,0.8]e^{[0.4,0.8]\text{л}i}, \\ [0.6,0.7]e^{[0.5,0.9]\text{л}i}\end{pmatrix}, \begin{pmatrix}0.6e^{(0.8)\text{л}i}, \\ 0.5e^{(0.9)\text{л}i}\end{pmatrix}\right) \\
ţ_5, \left(\begin{pmatrix}[0.3,0.7]e^{[0.6,0.7]\text{л}i}, \\ [0.5,0.6]e^{[0.5,0.8]\text{л}i}\end{pmatrix}, \begin{pmatrix}0.5e^{(0.6)\text{л}i}, \\ 0.6e^{(0.7)\text{л}i}\end{pmatrix}\right), \left(\begin{pmatrix}[0.2,0.7]e^{[0.6,0.8]\text{л}i}, \\ [0.5,0.6]e^{[0.6,0.9]\text{л}i}\end{pmatrix}, \begin{pmatrix}0.4e^{(0.9)\text{л}i}, \\ 0.2e^{(0.7)\text{л}i}\end{pmatrix}\right), \left(\begin{pmatrix}[0.4,0.8]e^{[0.5,0.7]\text{л}i}, \\ [0.4,0.6]e^{[0.5,0.7]\text{л}i}\end{pmatrix}, \begin{pmatrix}0.3e^{(0.6)\text{л}i}, \\ 0.4e^{(0.7)\text{л}i}\end{pmatrix}\right), \left(\begin{pmatrix}[0.7,0.8]e^{[0.3,0.4]\text{л}i}, \\ [0.4,0.8]e^{[0.6,0.9]\text{л}i}\end{pmatrix}, \begin{pmatrix}0.7e^{(0.9)\text{л}i}, \\ 0.5e^{(0.8)\text{л}i}\end{pmatrix}\right) \\
ţ_6, \left(\begin{pmatrix}[0.3,0.5]e^{[0.5,0.6]\text{л}i}, \\ [0.2,0.8]e^{[0.7,0.9]\text{л}i}\end{pmatrix}, \begin{pmatrix}0.6e^{(0.7)\text{л}i}, \\ 0.8e^{(0.9)\text{л}i}\end{pmatrix}\right), \left(\begin{pmatrix}[0.3,0.5]e^{[0.6,0.8]\text{л}i}, \\ [0.3,0.8]e^{[0.6,0.8]\text{л}i}\end{pmatrix}, \begin{pmatrix}0.4e^{(0.7)\text{л}i}, \\ 0.5e^{(0.9)\text{л}i}\end{pmatrix}\right), \left(\begin{pmatrix}[0.7,0.8]e^{[0.3,0.4]\text{л}i}, \\ [0.4,0.8]e^{[0.6,0.9]\text{л}i}\end{pmatrix}, \begin{pmatrix}0.7e^{(0.9)\text{л}i}, \\ 0.5e^{(0.8)\text{л}i}\end{pmatrix}\right), \left(\begin{pmatrix}[0.2,0.7]e^{[0.6,0.8]\text{л}i}, \\ [0.3,0.8]e^{[0.6,0.9]\text{л}i}\end{pmatrix}, \begin{pmatrix}0.4e^{(0.5)\text{л}i}, \\ 0.5e^{(0.9)\text{л}i}\end{pmatrix}\right)
\end{array} \right\}$$

Then the Cartesian product of securities and threats are as shown in Table 1.

**Table 1**
Cartesian product of web Services

| Ordered Pair | $u_1$ | $u_2$ | $u_3$ | $\lambda$ |
|---|---|---|---|---|
| $(ţ_1, ţ_1)$ | $\begin{pmatrix}[0.4,0.6]e^{[0.3,0.4]\text{л}i}, \\ [0.4,0.8]e^{[0.5,0.9]\text{л}i}\end{pmatrix}, \begin{pmatrix}0.3e^{(0.4)\text{л}i}, \\ 0.5e^{(0.8)\text{л}i}\end{pmatrix}$ | $\begin{pmatrix}[0.3,0.7]e^{[0.4,0.7]\text{л}i}, \\ [0.5,0.8]e^{[0.8,0.9]\text{л}i}\end{pmatrix}, \begin{pmatrix}0.3e^{(0.4)\text{л}i}, \\ 0.5e^{(0.8)\text{л}i}\end{pmatrix}$ | $\begin{pmatrix}[0.3,0.8]e^{[0.3,0.7]\text{л}i}, \\ [0.5,0.9]e^{[0.6,0.8]\text{л}i}\end{pmatrix}, \begin{pmatrix}0.4e^{(0.5)\text{л}i}, \\ 0.7e^{(0.9)\text{л}i}\end{pmatrix}$ | $\begin{pmatrix}[0.5,0.6]e^{[0.7,0.7]\text{л}i}, \\ [0.7,0.7]e^{[0.7,0.9]\text{л}i}\end{pmatrix}, \begin{pmatrix}0.5e^{(0.8)\text{л}i}, \\ 0.4e^{(0.7)\text{л}i}\end{pmatrix}$ |
| $(ţ_1, ţ_2)$ | $\begin{pmatrix}[0.4,0.6]e^{[0.3,0.4]\text{л}i}, \\ [0.4,0.8]e^{[0.6,0.9]\text{л}i}\end{pmatrix}, \begin{pmatrix}0.3e^{(0.4)\text{л}i}, \\ 0.5e^{(0.8)\text{л}i}\end{pmatrix}$ | $\begin{pmatrix}[0.3,0.7]e^{[0.4,0.7]\text{л}i}, \\ [0.5,0.8]e^{[0.8,0.9]\text{л}i}\end{pmatrix}, \begin{pmatrix}0.3e^{(0.4)\text{л}i}, \\ 0.5e^{(0.8)\text{л}i}\end{pmatrix}$ | $\begin{pmatrix}[0.3,0.7]e^{[0.4,0.7]\text{л}i}, \\ [0.5,0.8]e^{[0.8,0.9]\text{л}i}\end{pmatrix}, \begin{pmatrix}0.3e^{(0.4)\text{л}i}, \\ 0.5e^{(0.8)\text{л}i}\end{pmatrix}$ | $\begin{pmatrix}[0.3,0.7]e^{[0.4,0.7]\text{л}i}, \\ [0.5,0.8]e^{[0.8,0.9]\text{л}i}\end{pmatrix}, \begin{pmatrix}0.3e^{(0.5)\text{л}i}, \\ 0.5e^{(0.8)\text{л}i}\end{pmatrix}$ |
| $(ţ_1, ţ_3)$ | $\begin{pmatrix}[0.4,0.6]e^{[0.3,0.4]\text{л}i}, \\ [0.4,0.8]e^{[0.6,0.9]\text{л}i}\end{pmatrix}, \begin{pmatrix}0.3e^{(0.4)\text{л}i}, \\ 0.5e^{(0.9)\text{л}i}\end{pmatrix}$ | $\begin{pmatrix}[0.3,0.7]e^{[0.4,0.7]\text{л}i}, \\ [0.5,0.8]e^{[0.8,0.9]\text{л}i}\end{pmatrix}, \begin{pmatrix}0.3e^{(0.4)\text{л}i}, \\ 0.5e^{(0.8)\text{л}i}\end{pmatrix}$ | $\begin{pmatrix}[0.3,0.7]e^{[0.4,0.7]\text{л}i}, \\ [0.5,0.8]e^{[0.8,0.9]\text{л}i}\end{pmatrix}, \begin{pmatrix}0.3e^{(0.4)\text{л}i}, \\ 0.5e^{(0.8)\text{л}i}\end{pmatrix}$ | $\begin{pmatrix}[0.3,0.8]e^{[0.4,0.7]\text{л}i}, \\ [0.5,0.9]e^{[0.8,0.9]\text{л}i}\end{pmatrix}, \begin{pmatrix}0.3e^{(0.7)\text{л}i}, \\ 0.5e^{(0.6)\text{л}i}\end{pmatrix}$ |
| $(ţ_1, ţ_4)$ | $\begin{pmatrix}[0.4,0.6]e^{[0.3,0.4]\text{л}i}, \\ [0.4,0.8]e^{[0.6,0.9]\text{л}i}\end{pmatrix}, \begin{pmatrix}0.3e^{(0.4)\text{л}i}, \\ 0.5e^{(0.7)\text{л}i}\end{pmatrix}$ | $\begin{pmatrix}[0.3,0.7]e^{[0.4,0.7]\text{л}i}, \\ [0.5,0.8]e^{[0.8,0.9]\text{л}i}\end{pmatrix}, \begin{pmatrix}0.3e^{(0.4)\text{л}i}, \\ 0.5e^{(0.8)\text{л}i}\end{pmatrix}$ | $\begin{pmatrix}[0.4,0.6]e^{[0.3,0.4]\text{л}i}, \\ [0.4,0.8]e^{[0.6,0.9]\text{л}i}\end{pmatrix}, \begin{pmatrix}0.3e^{(0.4)\text{л}i}, \\ 0.5e^{(0.8)\text{л}i}\end{pmatrix}$ | $\begin{pmatrix}[0.3,0.7]e^{[0.4,0.8]\text{л}i}, \\ [0.5,0.8]e^{[0.6,0.9]\text{л}i}\end{pmatrix}, \begin{pmatrix}0.5e^{(0.7)\text{л}i}, \\ 0.4e^{(0.8)\text{л}i}\end{pmatrix}$ |
| $(ţ_1, ţ_5)$ | $\begin{pmatrix}[0.4,0.6]e^{[0.3,0.4]\text{л}i}, \\ [0.4,0.8]e^{[0.6,0.8]\text{л}i}\end{pmatrix}, \begin{pmatrix}0.3e^{(0.4)\text{л}i}, \\ 0.5e^{(0.8)\text{л}i}\end{pmatrix}$ | $\begin{pmatrix}[0.3,0.7]e^{[0.4,0.7]\text{л}i}, \\ [0.5,0.8]e^{[0.8,0.9]\text{л}i}\end{pmatrix}, \begin{pmatrix}0.3e^{(0.4)\text{л}i}, \\ 0.5e^{(0.8)\text{л}i}\end{pmatrix}$ | $\begin{pmatrix}[0.3,0.6]e^{[0.3,0.6]\text{л}i}, \\ [0.5,0.8]e^{[0.6,0.9]\text{л}i}\end{pmatrix}, \begin{pmatrix}0.4e^{(0.6)\text{л}i}, \\ 0.8e^{(0.9)\text{л}i}\end{pmatrix}$ | $\begin{pmatrix}[0.3,0.6]e^{[0.3,0.4]\text{л}i}, \\ [0.4,0.9]e^{[0.6,0.9]\text{л}i}\end{pmatrix}, \begin{pmatrix}0.3e^{(0.7)\text{л}i}, \\ 0.5e^{(0.8)\text{л}i}\end{pmatrix}$ |

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**Table 1**
Continued

| Ordered Pair | $u_1$ | $u_2$ | $u_3$ | $\lambda$ |
|---|---|---|---|---|
| $(ƫ_2, ƫ_1)$ | $\left([0.4,0.6]e^{[0.3,0.4]\pi i}, [0.4,0.7]e^{[0.6,0.9]\pi i}\right), \left(0.3e^{(0.4)\pi i}, 0.5e^{(0.8)\pi i}\right)$ | $\left([0.3,0.7]e^{[0.4,0.7]\pi i}, [0.5,0.8]e^{[0.8,0.9]\pi i}\right), \left(0.3e^{(0.4)\pi i}, 0.5e^{(0.8)\pi i}\right)$ | $\left([0.3,0.6]e^{[0.3,0.6]\pi i}, [0.5,0.8]e^{[0.6,0.9]\pi i}\right), \left(0.4e^{(0.6)\pi i}, 0.8e^{(0.9)\pi i}\right)$ | $\left([0.4,0.6]e^{[0.3,0.4]\pi i}, [0.4,0.7]e^{[0.6,0.9]\pi i}\right), \left(0.4e^{(0.5)\pi i}, 0.5e^{(0.8)\pi i}\right)$ |
| $(ƫ_2, ƫ_2)$ | $\left([0.4,0.6]e^{[0.3,0.4]\pi i}, [0.4,0.6]e^{[0.6,0.9]\pi i}\right), \left(0.3e^{(0.4)\pi i}, 0.5e^{(0.8)\pi i}\right)$ | $\left([0.3,0.7]e^{[0.4,0.7]\pi i}, [0.5,0.8]e^{[0.8,0.9]\pi i}\right), \left(0.3e^{(0.4)\pi i}, 0.5e^{(0.8)\pi i}\right)$ | $\left([0.4,0.6]e^{[0.3,0.4]\pi i}, [0.4,0.8]e^{[0.6,0.9]\pi i}\right), \left(0.3e^{(0.4)\pi i}, 0.5e^{(0.8)\pi i}\right)$ | $\left([0.3,0.6]e^{[0.4,0.7]\pi i}, [0.5,0.9]e^{[0.8,0.9]\pi i}\right), \left(0.3e^{(0.4)\pi i}, 0.4e^{(0.8)\pi i}\right)$ |
| $(ƫ_2, ƫ_3)$ | $\left([0.4,0.6]e^{[0.3,0.4]\pi i}, [0.4,0.8]e^{[0.6,0.9]\pi i}\right), \left(0.3e^{(0.4)\pi i}, 0.5e^{(0.7)\pi i}\right)$ | $\left([0.3,0.7]e^{[0.4,0.7]\pi i}, [0.5,0.8]e^{[0.8,0.9]\pi i}\right), \left(0.3e^{(0.4)\pi i}, 0.5e^{(0.8)\pi i}\right)$ | $\left([0.3,0.7]e^{[0.4,0.7]\pi i}, [0.5,0.8]e^{[0.8,0.9]\pi i}\right), \left(0.3e^{(0.4)\pi i}, 0.5e^{(0.8)\pi i}\right)$ | $\left([0.3,0.7]e^{[0.4,0.7]\pi i}, [0.5,0.8]e^{[0.8,0.9]\pi i}\right), \left(0.3e^{(0.4)\pi i}, 0.5e^{(0.8)\pi i}\right)$ |
| $(ƫ_2, ƫ_4)$ | $\left([0.4,0.6]e^{[0.3,0.4]\pi i}, [0.4,0.8]e^{[0.6,0.9]\pi i}\right), \left(0.3e^{(0.5)\pi i}, 0.5e^{(0.8)\pi i}\right)$ | $\left([0.3,0.7]e^{[0.4,0.7]\pi i}, [0.5,0.8]e^{[0.8,0.9]\pi i}\right), \left(0.3e^{(0.4)\pi i}, 0.5e^{(0.8)\pi i}\right)$ | $\left([0.3,0.7]e^{[0.4,0.7]\pi i}, [0.5,0.8]e^{[0.8,0.9]\pi i}\right), \left(0.3e^{(0.4)\pi i}, 0.4e^{(0.8)\pi i}\right)$ | $\left([0.3,0.6]e^{[0.4,0.6]\pi i}, [0.5,0.8]e^{[0.4,0.9]\pi i}\right), \left(0.3e^{(0.4)\pi i}, 0.5e^{(0.8)\pi i}\right)$ |
| $(ƫ_2, ƫ_5)$ | $\left([0.4,0.6]e^{[0.3,0.4]\pi i}, [0.4,0.8]e^{[0.6,0.9]\pi i}\right), \left(0.3e^{(0.4)\pi i}, 0.5e^{(0.8)\pi i}\right)$ | $\left([0.3,0.7]e^{[0.4,0.7]\pi i}, [0.5,0.8]e^{[0.8,0.9]\pi i}\right), \left(0.3e^{(0.4)\pi i}, 0.5e^{(0.8)\pi i}\right)$ | $\left([0.3,0.6]e^{[0.3,0.6]\pi i}, [0.5,0.8]e^{[0.6,0.9]\pi i}\right), \left(0.4e^{(0.6)\pi i}, 0.8e^{(0.9)\pi i}\right)$ | $\left([0.3,0.7]e^{[0.4,0.7]\pi i}, [0.5,0.8]e^{[0.8,0.9]\pi i}\right), \left(0.3e^{(0.6)\pi i}, 0.5e^{(0.9)\pi i}\right)$ |
| $(ƫ_2, ƫ_6)$ | $\left([0.4,0.6]e^{[0.3,0.4]\pi i}, [0.4,0.8]e^{[0.6,0.9]\pi i}\right), \left(0.3e^{(0.4)\pi i}, 0.5e^{(0.7)\pi i}\right)$ | $\left([0.3,0.7]e^{[0.4,0.7]\pi i}, [0.5,0.8]e^{[0.8,0.9]\pi i}\right), \left(0.3e^{(0.4)\pi i}, 0.5e^{(0.8)\pi i}\right)$ | $\left([0.3,0.7]e^{[0.4,0.7]\pi i}, [0.5,0.8]e^{[0.5,0.9]\pi i}\right), \left(0.3e^{(0.4)\pi i}, 0.6e^{(0.8)\pi i}\right)$ | $\left([0.4,0.6]e^{[0.3,0.4]\pi i}, [0.4,0.5]e^{[0.6,0.9]\pi i}\right), \left(0.3e^{(0.4)\pi i}, 0.3e^{(0.8)\pi i}\right)$ |
| $(ƫ_3, ƫ_1)$ | $\left([0.4,0.6]e^{[0.3,0.4]\pi i}, [0.4,0.8]e^{[0.6,0.9]\pi i}\right), \left(0.3e^{(0.4)\pi i}, 0.5e^{(0.6)\pi i}\right)$ | $\left([0.3,0.7]e^{[0.4,0.7]\pi i}, [0.5,0.8]e^{[0.8,0.9]\pi i}\right), \left(0.3e^{(0.4)\pi i}, 0.5e^{(0.8)\pi i}\right)$ | $\left([0.3,0.6]e^{[0.3,0.6]\pi i}, [0.5,0.7]e^{[0.6,0.9]\pi i}\right), \left(0.4e^{(0.6)\pi i}, 0.7e^{(0.9)\pi i}\right)$ | $\left([0.4,0.6]e^{[0.3,0.4]\pi i}, [0.4,0.9]e^{[0.6,0.9]\pi i}\right), \left(0.4e^{(0.4)\pi i}, 0.5e^{(0.8)\pi i}\right)$ |
| $(ƫ_3, ƫ_2)$ | $\left([0.4,0.6]e^{[0.3,0.4]\pi i}, [0.4,0.8]e^{[0.6,0.9]\pi i}\right), \left(0.3e^{(0.4)\pi i}, 0.5e^{(0.9)\pi i}\right)$ | $\left([0.3,0.7]e^{[0.4,0.7]\pi i}, [0.5,0.8]e^{[0.8,0.9]\pi i}\right), \left(0.3e^{(0.4)\pi i}, 0.5e^{(0.8)\pi i}\right)$ | $\left([0.3,0.7]e^{[0.4,0.7]\pi i}, [0.5,0.8]e^{[0.8,0.9]\pi i}\right), \left(0.3e^{(0.4)\pi i}, 0.5e^{(0.8)\pi i}\right)$ | $\left([0.3,0.6]e^{[0.4,0.7]\pi i}, [0.5,0.8]e^{[0.6,0.9]\pi i}\right), \left(0.3e^{(0.6)\pi i}, 0.5e^{(0.7)\pi i}\right)$ |
| $(ƫ_3, ƫ_3)$ | $\left([0.4,0.6]e^{[0.3,0.4]\pi i}, [0.4,0.8]e^{[0.6,0.9]\pi i}\right), \left(0.3e^{(0.4)\pi i}, 0.5e^{(0.8)\pi i}\right)$ | $\left([0.3,0.7]e^{[0.4,0.7]\pi i}, [0.5,0.8]e^{[0.8,0.9]\pi i}\right), \left(0.3e^{(0.4)\pi i}, 0.5e^{(0.8)\pi i}\right)$ | $\left([0.3,0.6]e^{[0.3,0.6]\pi i}, [0.5,0.8]e^{[0.6,0.9]\pi i}\right), \left(0.4e^{(0.6)\pi i}, 0.8e^{(0.9)\pi i}\right)$ | $\left([0.3,0.6]e^{[0.3,0.7]\pi i}, [0.4,0.7]e^{[0.4,0.9]\pi i}\right), \left(0.4e^{(0.4)\pi i}, 0.5e^{(0.9)\pi i}\right)$ |
| $(ƫ_3, ƫ_4)$ | $\left([0.4,0.6]e^{[0.3,0.6]\pi i}, [0.4,0.8]e^{[0.6,0.9]\pi i}\right), \left(0.3e^{(0.4)\pi i}, 0.6e^{(0.8)\pi i}\right)$ | $\left([0.3,0.7]e^{[0.4,0.7]\pi i}, [0.5,0.8]e^{[0.8,0.9]\pi i}\right), \left(0.3e^{(0.4)\pi i}, 0.5e^{(0.8)\pi i}\right)$ | $\left([0.3,0.7]e^{[0.4,0.7]\pi i}, [0.5,0.8]e^{[0.8,0.9]\pi i}\right), \left(0.3e^{(0.4)\pi i}, 0.5e^{(0.8)\pi i}\right)$ | $\left([0.5,0.6]e^{[0.3,0.7]\pi i}, [0.3,0.8]e^{[0.6,0.9]\pi i}\right), \left(0.3e^{(0.6)\pi i}, 0.5e^{(0.7)\pi i}\right)$ |
| $(ƫ_3, ƫ_5)$ | $\left([0.4,0.6]e^{[0.3,0.4]\pi i}, [0.4,0.8]e^{[0.6,0.9]\pi i}\right), \left(0.3e^{(0.4)\pi i}, 0.5e^{(0.7)\pi i}\right)$ | $\left([0.3,0.7]e^{[0.4,0.7]\pi i}, [0.5,0.8]e^{[0.8,0.9]\pi i}\right), \left(0.3e^{(0.4)\pi i}, 0.5e^{(0.8)\pi i}\right)$ | $\left([0.3,0.6]e^{[0.3,0.6]\pi i}, [0.5,0.8]e^{[0.6,0.9]\pi i}\right), \left(0.4e^{(0.6)\pi i}, 0.8e^{(0.9)\pi i}\right)$ | $\left([0.3,0.8]e^{[0.4,0.9]\pi i}, [0.4,0.8]e^{[0.6,0.9]\pi i}\right), \left(0.4e^{(0.5)\pi i}, 0.7e^{(0.8)\pi i}\right)$ |
| $(ƫ_3, ƫ_6)$ | $\left([0.4,0.6]e^{[0.3,0.4]\pi i}, [0.4,0.8]e^{[0.6,0.9]\pi i}\right), \left(0.3e^{(0.4)\pi i}, 0.5e^{(0.8)\pi i}\right)$ | $\left([0.3,0.7]e^{[0.4,0.7]\pi i}, [0.5,0.8]e^{[0.8,0.9]\pi i}\right), \left(0.3e^{(0.4)\pi i}, 0.5e^{(0.8)\pi i}\right)$ | $\left([0.4,0.6]e^{[0.3,0.4]\pi i}, [0.4,0.8]e^{[0.6,0.9]\pi i}\right), \left(0.3e^{(0.4)\pi i}, 0.5e^{(0.8)\pi i}\right)$ | $\left([0.3,0.7]e^{[0.4,0.8]\pi i}, [0.5,0.8]e^{[0.5,0.9]\pi i}\right), \left(0.4e^{(0.6)\pi i}, 0.5e^{(0.7)\pi i}\right)$ |

**Table 1**
Continued

| Ordered Pair | $\boldsymbol{u_1}$ | $\boldsymbol{u_2}$ | $\boldsymbol{u_3}$ | $\boldsymbol{\lambda}$ |
|---|---|---|---|---|
| $(ƫ_4, ƫ_1)$ | $\begin{pmatrix}[0.4,0.6]e^{[0.3,0.4]лi},\\ [0.4,0.8]e^{[0.6,0.9]лi}\end{pmatrix}, \begin{pmatrix}0.3e^{(0.4)лi},\\ 0.5e^{(0.8)лi}\end{pmatrix}$ | $\begin{pmatrix}[0.3,0.7]e^{[0.4,0.7]лi},\\ [0.5,0.8]e^{[0.8,0.9]лi}\end{pmatrix}, \begin{pmatrix}0.3e^{(0.4)лi},\\ 0.5e^{(0.8)лi}\end{pmatrix}$ | $\begin{pmatrix}[0.4,0.6]e^{[0.3,0.4]лi},\\ [0.4,0.8]e^{[0.6,0.9]лi}\end{pmatrix}, \begin{pmatrix}0.3e^{(0.4)лi},\\ 0.5e^{(0.8)лi}\end{pmatrix}$ | $\begin{pmatrix}[0.3,0.6]e^{[0.6,0.7]лi},\\ [0.5,0.7]e^{[0.7,0.9]лi}\end{pmatrix}, \begin{pmatrix}0.3e^{(0.6)лi},\\ 0.5e^{(0.7)лi}\end{pmatrix}$ |
| $(ƫ_4, ƫ_2)$ | $\begin{pmatrix}[0.4,0.6]e^{[0.3,0.4]лi},\\ [0.4,0.8]e^{[0.6,0.9]лi}\end{pmatrix}, \begin{pmatrix}0.3e^{(0.4)лi},\\ 0.4e^{(0.8)лi}\end{pmatrix}$ | $\begin{pmatrix}[0.3,0.7]e^{[0.4,0.7]лi},\\ [0.5,0.8]e^{[0.8,0.9]лi}\end{pmatrix}, \begin{pmatrix}0.3e^{(0.4)лi},\\ 0.5e^{(0.8)лi}\end{pmatrix}$ | $\begin{pmatrix}[0.3,0.6]e^{[0.3,0.6]лi},\\ [0.5,0.8]e^{[0.6,0.9]лi}\end{pmatrix}, \begin{pmatrix}0.4e^{(0.6)лi},\\ 0.8e^{(0.9)лi}\end{pmatrix}$ | $\begin{pmatrix}[0.3,0.7]e^{[0.4,0.7]лi},\\ [0.5,0.8]e^{[0.8,0.9]лi}\end{pmatrix}, \begin{pmatrix}0.3e^{(0.5)лi},\\ 0.5e^{(0.8)лi}\end{pmatrix}$ |
| $(ƫ_4, ƫ_3)$ | $\begin{pmatrix}[0.4,0.6]e^{[0.3,0.4]лi},\\ [0.4,0.8]e^{[0.6,0.9]лi}\end{pmatrix}, \begin{pmatrix}0.4e^{(0.4)лi},\\ 0.5e^{(0.8)лi}\end{pmatrix}$ | $\begin{pmatrix}[0.3,0.7]e^{[0.4,0.7]лi},\\ [0.5,0.8]e^{[0.8,0.9]лi}\end{pmatrix}, \begin{pmatrix}0.3e^{(0.4)лi},\\ 0.5e^{(0.8)лi}\end{pmatrix}$ | $\begin{pmatrix}[0.3,0.7]e^{[0.4,0.7]лi},\\ [0.5,0.8]e^{[0.8,0.9]лi}\end{pmatrix}, \begin{pmatrix}0.3e^{(0.4)лi},\\ 0.5e^{(0.8)лi}\end{pmatrix}$ | $\begin{pmatrix}[0.4,0.6]e^{[0.3,0.4]лi},\\ [0.4,0.8]e^{[0.6,0.9]лi}\end{pmatrix}, \begin{pmatrix}0.3e^{(0.4)лi},\\ 0.5e^{(0.8)лi}\end{pmatrix}$ |
| $(ƫ_4, ƫ_4)$ | $\begin{pmatrix}[0.4,0.6]e^{[0.3,0.4]лi},\\ [0.4,0.8]e^{[0.6,0.9]лi}\end{pmatrix}, \begin{pmatrix}0.3e^{(0.4)лi},\\ 0.7e^{(0.8)лi}\end{pmatrix}$ | $\begin{pmatrix}[0.3,0.7]e^{[0.4,0.7]лi},\\ [0.5,0.8]e^{[0.8,0.9]лi}\end{pmatrix}, \begin{pmatrix}0.3e^{(0.4)лi},\\ 0.5e^{(0.8)лi}\end{pmatrix}$ | $\begin{pmatrix}[0.3,0.6]e^{[0.3,0.6]лi},\\ [0.5,0.8]e^{[0.6,0.9]лi}\end{pmatrix}, \begin{pmatrix}0.4e^{(0.6)лi},\\ 0.8e^{(0.9)лi}\end{pmatrix}$ | $\begin{pmatrix}[0.3,0.7]e^{[0.4,0.7]лi},\\ [0.5,0.8]e^{[0.8,0.9]лi}\end{pmatrix}, \begin{pmatrix}0.3e^{(0.4)лi},\\ 0.4e^{(0.9)лi}\end{pmatrix}$ |
| $(ƫ_4, ƫ_5)$ | $\begin{pmatrix}[0.4,0.6]e^{[0.3,0.5]лi},\\ [0.4,0.7]e^{[0.6,0.9]лi}\end{pmatrix}, \begin{pmatrix}0.3e^{(0.4)лi},\\ 0.5e^{(0.8)лi}\end{pmatrix}$ | $\begin{pmatrix}[0.3,0.7]e^{[0.4,0.7]лi},\\ [0.5,0.8]e^{[0.8,0.9]лi}\end{pmatrix}, \begin{pmatrix}0.3e^{(0.4)лi},\\ 0.5e^{(0.8)лi}\end{pmatrix}$ | $\begin{pmatrix}[0.4,0.6]e^{[0.3,0.4]лi},\\ [0.4,0.8]e^{[0.6,0.9]лi}\end{pmatrix}, \begin{pmatrix}0.3e^{(0.4)лi},\\ 0.5e^{(0.8)лi}\end{pmatrix}$ | $\begin{pmatrix}[0.3,0.7]e^{[0.4,0.7]лi},\\ [0.5,0.8]e^{[0.8,0.8]лi}\end{pmatrix}, \begin{pmatrix}0.3e^{(0.6)лi},\\ 0.5e^{(0.8)лi}\end{pmatrix}$ |
| $(ƫ_4, ƫ_6)$ | $\begin{pmatrix}[0.4,0.6]e^{[0.3,0.4]лi},\\ [0.4,0.8]e^{[0.5,0.9]лi}\end{pmatrix}, \begin{pmatrix}0.3e^{(0.4)лi},\\ 0.5e^{(0.9)лi}\end{pmatrix}$ | $\begin{pmatrix}[0.3,0.7]e^{[0.4,0.7]лi},\\ [0.5,0.8]e^{[0.8,0.9]лi}\end{pmatrix}, \begin{pmatrix}0.3e^{(0.4)лi},\\ 0.5e^{(0.8)лi}\end{pmatrix}$ | $\begin{pmatrix}[0.3,0.6]e^{[0.3,0.6]лi},\\ [0.5,0.8]e^{[0.6,0.9]лi}\end{pmatrix}, \begin{pmatrix}0.4e^{(0.6)лi},\\ 0.8e^{(0.9)лi}\end{pmatrix}$ | $\begin{pmatrix}[0.3,0.7]e^{[0.4,0.8]лi},\\ [0.5,0.8]e^{[0.8,0.9]лi}\end{pmatrix}, \begin{pmatrix}0.4e^{(0.6)лi},\\ 0.5e^{(0.7)лi}\end{pmatrix}$ |
| $(ƫ_5, ƫ_1)$ | $\begin{pmatrix}[0.4,0.6]e^{[0.3,0.7]лi},\\ [0.4,0.8]e^{[0.7,0.9]лi}\end{pmatrix}, \begin{pmatrix}0.3e^{(0.4)лi},\\ 0.5e^{(0.8)лi}\end{pmatrix}$ | $\begin{pmatrix}[0.3,0.7]e^{[0.4,0.7]лi},\\ [0.5,0.8]e^{[0.8,0.9]лi}\end{pmatrix}, \begin{pmatrix}0.3e^{(0.4)лi},\\ 0.5e^{(0.8)лi}\end{pmatrix}$ | $\begin{pmatrix}[0.3,0.6]e^{[0.3,0.6]лi},\\ [0.5,0.8]e^{[0.6,0.9]лi}\end{pmatrix}, \begin{pmatrix}0.4e^{(0.6)лi},\\ 0.8e^{(0.9)лi}\end{pmatrix}$ | $\begin{pmatrix}[0.3,0.7]e^{[0.4,0.7]лi},\\ [0.5,0.8]e^{[0.8,0.9]лi}\end{pmatrix}, \begin{pmatrix}0.3e^{(0.4)лi},\\ 0.5e^{(0.8)лi}\end{pmatrix}$ |
| $(ƫ_5, ƫ_2)$ | $\begin{pmatrix}[0.4,0.6]e^{[0.3,0.4]лi},\\ [0.4,0.7]e^{[0.5,0.9]лi}\end{pmatrix}, \begin{pmatrix}0.5e^{(0.8)лi},\\ 0.5e^{(0.6)лi}\end{pmatrix}$ | $\begin{pmatrix}[0.3,0.7]e^{[0.4,0.7]лi},\\ [0.5,0.8]e^{[0.8,0.9]лi}\end{pmatrix}, \begin{pmatrix}0.3e^{(0.4)лi},\\ 0.5e^{(0.8)лi}\end{pmatrix}$ | $\begin{pmatrix}[0.3,0.7]e^{[0.4,0.7]лi},\\ [0.5,0.8]e^{[0.8,0.9]лi}\end{pmatrix}, \begin{pmatrix}0.3e^{(0.4)лi},\\ 0.5e^{(0.8)лi}\end{pmatrix}$ | $\begin{pmatrix}[0.3,0.7]e^{[0.4,0.7]лi},\\ [0.5,0.8]e^{[0.8,0.9]лi}\end{pmatrix}, \begin{pmatrix}0.3e^{(0.4)лi},\\ 0.5e^{(0.8)лi}\end{pmatrix}$ |
| $(ƫ_5, ƫ_3)$ | $\begin{pmatrix}[0.4,0.6]e^{[0.3,0.4]лi},\\ [0.4,0.8]e^{[0.4,0.9]лi}\end{pmatrix}, \begin{pmatrix}0.3e^{(0.4)лi},\\ 0.5e^{(0.8)лi}\end{pmatrix}$ | $\begin{pmatrix}[0.3,0.7]e^{[0.4,0.7]лi},\\ [0.5,0.8]e^{[0.8,0.9]лi}\end{pmatrix}, \begin{pmatrix}0.3e^{(0.4)лi},\\ 0.5e^{(0.8)лi}\end{pmatrix}$ | $\begin{pmatrix}[0.3,0.7]e^{[0.4,0.7]лi},\\ [0.5,0.8]e^{[0.8,0.9]лi}\end{pmatrix}, \begin{pmatrix}0.3e^{(0.4)лi},\\ 0.5e^{(0.8)лi}\end{pmatrix}$ | $\begin{pmatrix}[0.3,0.7]e^{[0.4,0.7]лi},\\ [0.5,0.8]e^{[0.8,0.9]лi}\end{pmatrix}, \begin{pmatrix}0.3e^{(0.4)лi},\\ 0.5e^{(0.8)лi}\end{pmatrix}$ |
| $(ƫ_5, ƫ_4)$ | $\begin{pmatrix}[0.4,0.6]e^{[0.3,0.4]лi},\\ [0.4,0.7]e^{[0.6,0.9]лi}\end{pmatrix}, \begin{pmatrix}0.3e^{(0.4)лi},\\ 0.5e^{(0.8)лi}\end{pmatrix}$ | $\begin{pmatrix}[0.3,0.7]e^{[0.4,0.7]лi},\\ [0.5,0.8]e^{[0.8,0.9]лi}\end{pmatrix}, \begin{pmatrix}0.3e^{(0.4)лi},\\ 0.5e^{(0.8)лi}\end{pmatrix}$ | $\begin{pmatrix}[0.3,0.6]e^{[0.3,0.6]лi},\\ [0.5,0.8]e^{[0.6,0.9]лi}\end{pmatrix}, \begin{pmatrix}0.4e^{(0.6)лi},\\ 0.8e^{(0.9)лi}\end{pmatrix}$ | $\begin{pmatrix}[0.3,0.7]e^{[0.4,0.7]лi},\\ [0.5,0.8]e^{[0.8,0.9]лi}\end{pmatrix}, \begin{pmatrix}0.3e^{(0.4)лi},\\ 0.5e^{(0.8)лi}\end{pmatrix}$ |
| $(ƫ_5, ƫ_5)$ | $\begin{pmatrix}[0.4,0.6]e^{[0.3,0.4]лi},\\ [0.4,0.8]e^{[0.6,0.9]лi}\end{pmatrix}, \begin{pmatrix}0.5e^{(0.7)лi},\\ 0.5e^{(0.8)лi}\end{pmatrix}$ | $\begin{pmatrix}[0.3,0.7]e^{[0.4,0.7]лi},\\ [0.5,0.8]e^{[0.8,0.9]лi}\end{pmatrix}, \begin{pmatrix}0.3e^{(0.4)лi},\\ 0.5e^{(0.8)лi}\end{pmatrix}$ | $\begin{pmatrix}[0.3,0.6]e^{[0.3,0.6]лi},\\ [0.5,0.8]e^{[0.6,0.9]лi}\end{pmatrix}, \begin{pmatrix}0.4e^{(0.6)лi},\\ 0.8e^{(0.9)лi}\end{pmatrix}$ | $\begin{pmatrix}[0.3,0.7]e^{[0.4,0.7]лi},\\ [0.5,0.8]e^{[0.8,0.9]лi}\end{pmatrix}, \begin{pmatrix}0.3e^{(0.4)лi},\\ 0.5e^{(0.8)лi}\end{pmatrix}$ |
| $(ƫ_5, ƫ_6)$ | $\begin{pmatrix}[0.4,0.6]e^{[0.3,0.7]лi},\\ [0.5,0.8]e^{[0.6,0.9]лi}\end{pmatrix}, \begin{pmatrix}0.3e^{(0.4)лi},\\ 0.5e^{(0.8)лi}\end{pmatrix}$ | $\begin{pmatrix}[0.3,0.7]e^{[0.4,0.7]лi},\\ [0.5,0.8]e^{[0.8,0.9]лi}\end{pmatrix}, \begin{pmatrix}0.3e^{(0.4)лi},\\ 0.5e^{(0.8)лi}\end{pmatrix}$ | $\begin{pmatrix}[0.3,0.6]e^{[0.3,0.6]лi},\\ [0.5,0.8]e^{[0.6,0.9]лi}\end{pmatrix}, \begin{pmatrix}0.4e^{(0.6)лi},\\ 0.8e^{(0.9)лi}\end{pmatrix}$ | $\begin{pmatrix}[0.3,0.7]e^{[0.6,0.7]лi},\\ [0.5,0.6]e^{[0.8,0.9]лi}\end{pmatrix}, \begin{pmatrix}0.3e^{(0.4)лi},\\ 0.5e^{(0.8)лi}\end{pmatrix}$ |

**Table 1**
Continued

| Ordered Pair | $\boldsymbol{u_1}$ | $\boldsymbol{u_2}$ | $\boldsymbol{u_3}$ | $\boldsymbol{\lambda}$ |
|---|---|---|---|---|
| $(\text{ţ}_6, \text{ţ}_1)$ | $\begin{pmatrix}[0.4,0.6]e^{[0.3,0.4]\text{л}i}, \\ [0.4,0.8]e^{[0.6,0.9]\text{л}i}\end{pmatrix}, \begin{pmatrix}0.3e^{(0.4)\text{л}i}, \\ 0.5e^{(0.8)\text{л}i}\end{pmatrix}$ | $\begin{pmatrix}[0.3,0.7]e^{[0.4,0.7]\text{л}i}, \\ [0.5,0.8]e^{[0.8,0.9]\text{л}i}\end{pmatrix}, \begin{pmatrix}0.3e^{(0.4)\text{л}i}, \\ 0.5e^{(0.8)\text{л}i}\end{pmatrix}$ | $\begin{pmatrix}[0.3,0.6]e^{[0.3,0.6]\text{л}i}, \\ [0.5,0.8]e^{[0.6,0.9]\text{л}i}\end{pmatrix}, \begin{pmatrix}0.4e^{(0.6)\text{л}i}, \\ 0.8e^{(0.9)\text{л}i}\end{pmatrix}$ | $\begin{pmatrix}[0.3,0.7]e^{[0.4,0.9]\text{л}i}, \\ [0.5,0.8]e^{[0.8,0.9]\text{л}i}\end{pmatrix}, \begin{pmatrix}0.3e^{(0.5)\text{л}i}, \\ 0.5e^{(0.8)\text{л}i}\end{pmatrix}$ |
| $(\text{ţ}_6, \text{ţ}_2)$ | $\begin{pmatrix}[0.5,0.6]e^{[0.3,0.6]\text{л}i}, \\ [0.4,0.7]e^{[0.3,0.9]\text{л}i}\end{pmatrix}, \begin{pmatrix}0.3e^{(0.6)\text{л}i}, \\ 0.3e^{(0.8)\text{л}i}\end{pmatrix}$ | $\begin{pmatrix}[0.3,0.7]e^{[0.4,0.7]\text{л}i}, \\ [0.5,0.8]e^{[0.8,0.9]\text{л}i}\end{pmatrix}, \begin{pmatrix}0.3e^{(0.4)\text{л}i}, \\ 0.5e^{(0.8)\text{л}i}\end{pmatrix}$ | $\begin{pmatrix}[0.3,0.7]e^{[0.4,0.7]\text{л}i}, \\ [0.5,0.8]e^{[0.8,0.9]\text{л}i}\end{pmatrix}, \begin{pmatrix}0.3e^{(0.4)\text{л}i}, \\ 0.5e^{(0.8)\text{л}i}\end{pmatrix}$ | $\begin{pmatrix}[0.3,0.7]e^{[0.4,0.7]\text{л}i}, \\ [0.5,0.8]e^{[0.8,0.9]\text{л}i}\end{pmatrix}, \begin{pmatrix}0.3e^{(0.4)\text{л}i}, \\ 0.5e^{(0.8)\text{л}i}\end{pmatrix}$ |
| $(\text{ţ}_6, \text{ţ}_3)$ | $\begin{pmatrix}[0.4,0.6]e^{[0.3,0.4]\text{л}i}, \\ [0.4,0.6]e^{[0.6,0.8]\text{л}i}\end{pmatrix}, \begin{pmatrix}0.5e^{(0.7)\text{л}i}, \\ 0.5e^{(0.8)\text{л}i}\end{pmatrix}$ | $\begin{pmatrix}[0.3,0.7]e^{[0.4,0.7]\text{л}i}, \\ [0.5,0.8]e^{[0.8,0.9]\text{л}i}\end{pmatrix}, \begin{pmatrix}0.3e^{(0.4)\text{л}i}, \\ 0.5e^{(0.8)\text{л}i}\end{pmatrix}$ | $\begin{pmatrix}[0.3,0.6]e^{[0.3,0.6]\text{л}i}, \\ [0.5,0.8]e^{[0.6,0.9]\text{л}i}\end{pmatrix}, \begin{pmatrix}0.4e^{(0.6)\text{л}i}, \\ 0.8e^{(0.9)\text{л}i}\end{pmatrix}$ | $\begin{pmatrix}[0.3,0.7]e^{[0.4,0.7]\text{л}i}, \\ [0.5,0.6]e^{[0.5,0.9]\text{л}i}\end{pmatrix}, \begin{pmatrix}0.3e^{(0.4)\text{л}i}, \\ 0.5e^{(0.8)\text{л}i}\end{pmatrix}$ |
| $(\text{ţ}_6, \text{ţ}_4)$ | $\begin{pmatrix}[0.4,0.6]e^{[0.3,0.4]\text{л}i}, \\ [0.4,0.7]e^{[0.6,0.9]\text{л}i}\end{pmatrix}, \begin{pmatrix}0.3e^{(0.4)\text{л}i}, \\ 0.5e^{(0.7)\text{л}i}\end{pmatrix}$ | $\begin{pmatrix}[0.3,0.7]e^{[0.4,0.7]\text{л}i}, \\ [0.5,0.8]e^{[0.8,0.9]\text{л}i}\end{pmatrix}, \begin{pmatrix}0.3e^{(0.4)\text{л}i}, \\ 0.5e^{(0.8)\text{л}i}\end{pmatrix}$ | $\begin{pmatrix}[0.3,0.6]e^{[0.3,0.6]\text{л}i}, \\ [0.5,0.8]e^{[0.6,0.9]\text{л}i}\end{pmatrix}, \begin{pmatrix}0.4e^{(0.6)\text{л}i}, \\ 0.8e^{(0.9)\text{л}i}\end{pmatrix}$ | $\begin{pmatrix}[0.3,0.7]e^{[0.4,0.7]\text{л}i}, \\ [0.5,0.8]e^{[0.8,0.9]\text{л}i}\end{pmatrix}, \begin{pmatrix}0.3e^{(0.6)\text{л}i}, \\ 0.5e^{(0.7)\text{л}i}\end{pmatrix}$ |
| $(\text{ţ}_6, \text{ţ}_5)$ | $\begin{pmatrix}[0.4,0.6]e^{[0.3,0.6]\text{л}i}, \\ [0.5,0.8]e^{[0.6,0.9]\text{л}i}\end{pmatrix}, \begin{pmatrix}0.3e^{(0.4)\text{л}i}, \\ 0.5e^{(0.8)\text{л}i}\end{pmatrix}$ | $\begin{pmatrix}[0.3,0.7]e^{[0.4,0.7]\text{л}i}, \\ [0.5,0.8]e^{[0.8,0.9]\text{л}i}\end{pmatrix}, \begin{pmatrix}0.3e^{(0.4)\text{л}i}, \\ 0.5e^{(0.8)\text{л}i}\end{pmatrix}$ | $\begin{pmatrix}[0.3,0.7]e^{[0.4,0.7]\text{л}i}, \\ [0.5,0.8]e^{[0.8,0.9]\text{л}i}\end{pmatrix}, \begin{pmatrix}0.3e^{(0.4)\text{л}i}, \\ 0.5e^{(0.8)\text{л}i}\end{pmatrix}$ | $\begin{pmatrix}[0.3,0.7]e^{[0.6,0.7]\text{л}i}, \\ [0.5,0.7]e^{[0.8,0.9]\text{л}i}\end{pmatrix}, \begin{pmatrix}0.3e^{(0.6)\text{л}i}, \\ 0.5e^{(0.8)\text{л}i}\end{pmatrix}$ |
| $(\text{ţ}_6, \text{ţ}_6)$ | $\begin{pmatrix}[0.3,0.6]e^{[0.3,0.4]\text{л}i}, \\ [0.4,0.8]e^{[0.6,0.8]\text{л}i}\end{pmatrix}, \begin{pmatrix}0.3e^{(0.4)\text{л}i}, \\ 0.5e^{(0.7)\text{л}i}\end{pmatrix}$ | $\begin{pmatrix}[0.3,0.7]e^{[0.4,0.7]\text{л}i}, \\ [0.5,0.8]e^{[0.8,0.9]\text{л}i}\end{pmatrix}, \begin{pmatrix}0.3e^{(0.4)\text{л}i}, \\ 0.5e^{(0.8)\text{л}i}\end{pmatrix}$ | $\begin{pmatrix}[0.3,0.6]e^{[0.3,0.6]\text{л}i}, \\ [0.5,0.8]e^{[0.6,0.9]\text{л}i}\end{pmatrix}, \begin{pmatrix}0.4e^{(0.6)\text{л}i}, \\ 0.8e^{(0.9)\text{л}i}\end{pmatrix}$ | $\begin{pmatrix}[0.3,0.8]e^{[0.4,0.7]\text{л}i}, \\ [0.5,0.9]e^{[0.5,0.9]\text{л}i}\end{pmatrix}, \begin{pmatrix}0.4e^{(0.4)\text{л}i}, \\ 0.7e^{(0.8)\text{л}i}\end{pmatrix}$ |

The Cartesian product of two CCuqROFSs is shown in the table above. To find the score function firstly we convert a complex number from exponential form to rectangular form $a + ib$, we can use the following formula. i.e., $a + ib = re^{\pi i\theta},\ as\ r = \sqrt{a^2 + b^2}\ and\ e^{\pi i\theta} = \cos(\theta) + isin(\theta)$, Then $a = rcos\pi(\theta), b = sin\pi(\theta)$. When we convert a complex number from exponential form to rectangular form, the angle $\pi$ represents the angle that the complex number makes with the positive real axis in the complex plane. This angle can be measured in radians or degrees. Take modulus after converting polar form to standard form. These procedures apply to the M and NM. After all this process, apply to M and NM score formula to $\frac{1}{2}\left|\left(\left[\left(\alpha_p^{-q}(\text{ţ}) - \alpha_n^{-q}(\text{ţ})\right)\right] + \left[\left(\beta_p^{-q}(\text{ţ}) - \beta_n^{-q}(\text{ţ})\right)\right]\right) + \left(\left[\left(\alpha_p^{+q}(\text{ţ}) - \alpha_n^{+q}(\text{ţ})\right)\right] + \left[\left(\beta_p^{+q}(\text{ţ}) - \beta_n^{+q}(\text{ţ})\right)\right]\right) + \left[\left(\alpha_p^{q}(\text{ţ}) - \alpha_n^{q}(\text{ţ})\right) + \left(\beta_p^{q}(\text{ţ}) + \beta_n^{q}(\text{ţ})\right)\right]\right|$ as shown in Table 2.

**Table 2**
Score value of web browser

| Ordered Pair | $\check{u}_1$ | $\check{u}_2$ | $\check{u}_3$ | $\boldsymbol{\lambda}$ |
|---|---|---|---|---|
| $(\text{ţ}_1, \text{ţ}_1)$ | 0.23 | 0.37 | 0.18 | 0.15 |
| $(\text{ţ}_1, \text{ţ}_2)$ | 0.20 | 0.23 | 0.07 | 0.18 |
| $(\text{ţ}_1, \text{ţ}_3)$ | 0.17 | 0.39 | 0.18 | 0.13 |
| $(\text{ţ}_1, \text{ţ}_4)$ | 0.34 | 0.31 | 0.18 | 0.11 |
| $(\text{ţ}_1, \text{ţ}_5)$ | 0.28 | 0.22 | 0.18 | 0.21 |
| $(\text{ţ}_1, \text{ţ}_6)$ | 0.27 | 0.22 | 0.16 | 0.20 |
| $(\text{ţ}_2, \text{ţ}_1)$ | 0.15 | 0.23 | 0.07 | 0.10 |

**Table 2**
Continued

| Ordered Pair | $\check{u}_1$ | $\check{u}_2$ | $\check{u}_3$ | $\lambda$ |
|---|---|---|---|---|
| $(ţ_2, ţ_2)$ | 0.23 | 0.24 | 0.10 | 0.20 |
| $(ţ_2, ţ_3)$ | 0.17 | 0.22 | 0.15 | 0.21 |
| $(ţ_2, ţ_4)$ | 0.01 | 0.22 | 0.07 | 0.21 |
| $(ţ_2, ţ_5)$ | 0.22 | 0.15 | 0.17 | 0.23 |
| $(ţ_2, ţ_6)$ | 0.11 | 0.21 | 0.17 | 0.25 |
| $(ţ_3, ţ_1)$ | 0.28 | 0.18 | 0.31 | 0.13 |
| $(ţ_3, ţ_2)$ | 0.23 | 0.11 | 0.18 | 0.21 |
| $(ţ_3, ţ_3)$ | 0.36 | 0.20 | 0.31 | 0.27 |
| $(ţ_3, ţ_4)$ | 0.29 | 0.31 | 0.18 | 0.26 |
| $(ţ_3, ţ_5)$ | 0.30 | 0.24 | 0.29 | 0.20 |
| $(ţ_3, ţ_6)$ | 0.30 | 0.24 | 0.29 | 0.20 |
| $(ţ_4, ţ_1)$ | 0.28 | 0.19 | 0.18 | 0.20 |
| $(ţ_4, ţ_2)$ | 0.17 | 0.22 | 0.07 | 0.23 |
| $(ţ_4, ţ_3)$ | 0.29 | 0.19 | 0.13 | 0.26 |
| $(ţ_4, ţ_4)$ | 0. 34 | 0.31 | 0.13 | 0.21 |
| $(ţ_4, ţ_5)$ | 0.27 | 0.22 | 0.16 | 0.28 |
| $(ţ_4, ţ_6)$ | 0.27 | 0.20 | 0.16 | 0.08 |
| $(ţ_5, ţ_1)$ | 0.27 | 0.22 | 0.16 | 0.20 |
| $(ţ_5, ţ_2)$ | 0.11 | 0.21 | 0.12 | 0.23 |
| $(ţ_5, ţ_3)$ | 0.30 | 0.24 | 0.16 | 0.20 |
| $(ţ_5, ţ_4)$ | 0.28 | 0.22 | 0.18 | 0.20 |
| $(ţ_5, ţ_5)$ | 0.30 | 0.25 | 0.29 | 0.25 |
| $(ţ_5, ţ_6)$ | 0.34 | 0.25 | 0.28 | 0.25 |
| $(ţ_6, ţ_1)$ | 0.34 | 0.25 | 0.06 | 0.15 |
| $(ţ_6, ţ_2)$ | 0.20 | 0.22 | 0.30 | 0.25 |
| $(ţ_6, ţ_3)$ | 0.10 | 0.21 | 0.22 | 0.25 |
| $(ţ_6, ţ_4)$ | 0.20 | 0.12 | 0.28 | 0.20 |
| $(ţ_6, ţ_5)$ | 0.30 | 0.25 | 0.18 | 0.25 |
| $(ţ_6, ţ_6)$ | 0.30 | 0.22 | 0.08 | 0.25 |

To find the best web services, we must first get the highest numerical degree in each row while ignoring the last column. The last column is the general belongingness of each web services. To calculate the score for each web service, we need to multiply the highest numerical degree in each row by the desired value of $\lambda$, and then add up these products. The resulting score represents the overall quality of the web service. The highest scoring web service is the one with the highest score value. This approach allows us to identify the web service that best meets our needs based on our specific criteria and priorities. Now, calculate the score function in Table 3.

**Table 3**
Grade Table of the score function

| Ordered pair | $(ţ_1, ţ_1)$ | $(ţ_1, ţ_2)$ | $(ţ_1, ţ_3)$ | $(ţ_1, ţ_4)$ | $(ţ_1, ţ_5)$ | $(ţ_1, ţ_6)$ | $(ţ_2, ţ_1)$ | $(ţ_2, ţ_2)$ | $(ţ_2, ţ_3)$ |
|---|---|---|---|---|---|---|---|---|---|
| $\check{u}_i$ | $\check{u}_2$ | $\check{u}_2$ | $\check{u}_2$ | $\check{u}_1$ | $\check{u}_1$ | $\check{u}_1$ | $\check{u}_2$ | $\check{u}_2$ | $\check{u}_2$ |
| Highest degree | × | 0.23 | 0.39 | 0.34 | 0.28 | 0.27 | 0.23 | × | 0.22 |
| $\lambda$ | × | 0.18 | 0.13 | 0.11 | 0.21 | 0.20 | 0.10 | × | 0.21 |

**Table 3**
Continued

| Ordered pair | $(ţ_2, ţ_4)$ | $(ţ_2, ţ_5)$ | $(ţ_2, ţ_6)$ | $(ţ_3, ţ_1)$ | $(ţ_3, ţ_2)$ | $(ţ_3, ţ_3)$ | $(ţ_3, ţ_4)$ | $(ţ_3, ţ_5)$ | $(ţ_3, ţ_6)$ |
|---|---|---|---|---|---|---|---|---|---|
| $ǔ_i$ | $ǔ_2$ | $ǔ_1$ | $ǔ_2$ | $ǔ_3$ | $ǔ_1$ | $ǔ_1$ | $ǔ_2$ | $ǔ_1$ | $ǔ_1$ |
| Ordered pair | $(ţ_4, ţ_1)$ | $(ţ_4, ţ_2)$ | $(ţ_4, ţ_3)$ | $(ţ_4, ţ_4)$ | $(ţ_4, ţ_5)$ | $(ţ_4, ţ_6)$ | $(ţ_5, ţ_1)$ | $(ţ_5, ţ_2)$ | $(ţ_5, ţ_3)$ |
| Highest degree | 0.22 | 0.22 | 0.21 | 0.31 | 0.23 | × | 0.31 | 0.30 | 0.30 |
| $\boldsymbol{\lambda}$ | 0.21 | 0.23 | 0.25 | 0.13 | 0.21 | × | 0.26 | 0.20 | 0.20 |
| Ordered pair | $(ţ_4, ţ_1)$ | $(ţ_4, ţ_2)$ | $(ţ_4, ţ_3)$ | $(ţ_4, ţ_4)$ | $(ţ_4, ţ_5)$ | $(ţ_4, ţ_6)$ | $(ţ_5, ţ_1)$ | $(ţ_5, ţ_2)$ | $(ţ_5, ţ_3)$ |
| $ǔ_i$ | $ǔ_1$ | $ǔ_2$ | $ǔ_1$ | $ǔ_2$ | $ǔ_1$ | $ǔ_1$ | $ǔ_1$ | $ǔ_2$ | $ǔ_1$ |
| Highest degree | 0.28 | 0.22 | 0.29 | × | 0.27 | 0.27 | 0.27 | 0.21 | 0.30 |
| $\boldsymbol{\lambda}$ | 0.20 | 0.23 | 0.26 | × | 0.28 | 0.08 | 0.20 | 0.23 | 0.20 |
| Ordered pair | $(ţ_5, ţ_4)$ | $(ţ_5, ţ_5)$ | $(ţ_5, ţ_6)$ | $(ţ_6, ţ_1)$ | $(ţ_6, ţ_2)$ | $(ţ_6, ţ_3)$ | $(ţ_6, ţ_4)$ | $(ţ_6, ţ_5)$ | $(ţ_6, ţ_6)$ |
| $ǔ_i$ | $ǔ_1$ | $ǔ_1$ | $ǔ_1$ | $ǔ_1$ | $ǔ_3$ | $ǔ_3$ | $ǔ_3$ | $ǔ_1$ | $ǔ_1$ |
| Highest degree | 0.28 | × | 0.34 | 0.34 | 0.30 | 0.22 | 0.28 | 0.30 | × |
| $\boldsymbol{\lambda}$ | 0.20 | × | 0.25 | 0.15 | 0.25 | 0.25 | 0.20 | 0.25 | × |

$$S(\mathbf{ǔ_1}) = (0.34 \times 0.11) + (0.28 \times 0.21) + (0.27 \times 0.20) + (0.22 \times 0.23) + (0.23 \times 0.21) + (0.30 \times 0.20) + (0.30 \times 0.20) + (0.28 \times 0.20) + (0.29 \times 0.26) + (0.27 \times 0.28) + (0.27 \times 0.08) + (0.27 \times 0.20) + (0.30 \times 0.20) + (0.28 \times 0.20)) + (0.34 \times 0.25) + (0.34 \times 0.15) + (0.30 \times 0.25) = 0.98$$

$$S(\mathbf{ǔ_2}) = (0.23 \times 0.18) + (0.39 \times 0.13) + (0.23 \times 0.10) + (0.22 \times 0.21) + (0.22 \times 0.21) + (0.21 \times 0.25) + (0.31 \times 0.26) + (0.22 \times 0.23) + (0.21 \times 0.23) = 0.44$$

$$S(\mathbf{ǔ_3}) = (0.31 \times 0.13) + (0.30 \times 0.25) + (0.22 \times 0.25) = 0.18$$

Web services metadata exchange is a type of web service that is considered to be highly efficient compared to other web services. This is due to its ability to provide a standardized way of describing the functionality of a web service, making it easier for users to discover and consume web services. By using metadata to describe the functionality of a web service, clients can dynamically invoke web services and easily integrate them into their applications, without the need for manual intervention. This efficiency is further increased by the fact that web service metadata can be easily accessed and utilized by different systems, regardless of their programming languages or platforms. Additionally, web services metadata exchange provides a way for web services to communicate with each other, enabling the development of more complex and integrated systems.

## 5. Comparative Analysis

In this part, we compare the presented structure of CCuqROFR with some other pre-existing structures such as FR, CuFR, CCuFR, IFR, CuIFR, CCuIFR, PyFR, CuPyFR, CCuPyFR, and qROFR.

### *5.1 Comparison of FR, CuFR, and CCuFR with CCuqROFRs*

The only discussion of the M degree with only one dimension is found in the FR and CuFR structures. They are unable of resolving the complex issue. The CuFR expressed the M degree in both the present and the future, whereas the FR simply discussed the present aspect of the M degree. The periodicity of these structures cannot be modelled. The CCuFR structures only mention M level, which solely demonstrates how well security measures counter threats. As a result, they are unable to provide a comprehensive solution to the issue. The levels, such as M and NM, are examined by the

CCuqROFR using a complex number. They have the ability to deal with periodicity. We take into account the two CCuFSs listed below, $j$ and $\mathbb{H}$, which stand in for the threat and security sets, respectively.

$$\text{j} = \left\{\begin{array}{c} \left(ETC, \left([0.4,0.6]e^{[0.3,0.6]\text{л}i}\right), \left(0.3e^{(0.4)\text{л}i}\right)\right), \left(\text{MMI}, \left([0.6,0.7]e^{[0.5,0.9]\text{л}i}\right), \left(0.4e^{(0.6)\text{л}i}\right)\right), \\ \left(\text{ABP}, \left([0.5,0.8]e^{[0.4,0.7]\text{л}i}\right), \left(0.5e^{(0.7)\text{л}i}\right)\right), \left(\text{TE}, \left([0.4,0.9]e^{[0.2,0.8]\text{л}i}\right), \left(0.1e^{(0.9)\text{л}i}\right)\right), \\ \left(\text{MC}, \left([0.3,0.6]e^{[0.3,0.7]\text{л}i}\right), \left(0.3e^{(0.8)\text{л}i}\right)\right), \left(\text{SV}, \left([0.2,0.3]e^{[0.3,0.5]\text{л}i}\right), \left(0.7e^{(0.8)\text{л}i}\right)\right), \end{array}\right\}$$

$$\mathbb{H} = \left\{\begin{array}{c} \left(VW, \left([0.7,0.8]e^{[0.3,0.4]\text{л}i}\right), \left(0.7e^{(0.9)\text{л}i}\right)\right), \left(\text{RW}, \left([0.5,0.7]e^{[0.4,0.5]\text{л}i}\right), \left(0.6e^{(0.8)\text{л}i}\right)\right), \\ \left(\text{CFI}, \left([0.3,0.7]e^{[0.6,0.8]\text{л}i}\right), \left(0.4e^{(0.7)\text{л}i}\right)\right), \left(\text{CF}, \left([0.4,0.8]e^{[0.5,0.7]\text{л}i}\right), \left(0.3e^{(0.6)\text{л}i}\right)\right), \\ \left(\text{BAC}, \left([0.3,0.7]e^{[0.6,0.7]\text{л}i}\right), \left(0.5e^{(0.6)\text{л}i}\right)\right), \left(\text{CSS}, \left([0.3,0.5]e^{[0.5,0.6]\text{л}i}\right), \left(0.6e^{(0.7)\text{л}i}\right)\right) \end{array}\right\}$$

Then the Cartesian product of j and $\mathbb{H}$ are as follows,

$$\text{j} \times \mathbb{H} = \left\{\begin{array}{c}
\left((ETC, VW), \left([0.4,0.6]e^{[0.3,0.4]\text{л}i}\right), \left(0.3e^{(0.4)\text{л}i}\right)\right) \left((ETC, RW), \left([0.4,0.6]e^{[0.3,0.5]\text{л}i}\right), \left(0.3e^{(0.4)\text{л}i}\right)\right), \\
, \left((ETC, CFI), \left([0.3,0.6]e^{[0.3,0.6]\text{л}i}\right), \left(0.3e^{(0.4)\text{л}i}\right)\right), \left((ETC, CF), \left([0.4,0.6]e^{[0.3,0.6]\text{л}i}\right), \left(0.3e^{(0.4)\text{л}i}\right)\right), \\
\left((ETC, BAC), \left([0.3,0.6]e^{[0.3,0.6]\text{л}i}\right), \left(0.3e^{(0.4)\text{л}i}\right)\right), \left((ETC, CSS), \left([0.3,0.5]e^{[0.3,0.6]\text{л}i}\right), \left(0.3e^{(0.4)\text{л}i}\right)\right), \\
\left((\text{MMI}, VW)\left([0.6,0.7]e^{[0.3,0.4]\text{л}i}\right), \left(0.4e^{(0.6)\text{л}i}\right)\right), \left((\text{MMI}, \text{RW})\left([0.5,0.7]e^{[0.4,0.5]\text{л}i}\right), \left(0.4e^{(0.6)\text{л}i}\right)\right), \\
\left((\text{MMI}, \text{CFI}), \left([0.3,0.7]e^{[0.5,0.8]\text{л}i}\right), \left(0.4e^{(0.6)\text{л}i}\right)\right), \left((\text{MMI}, \text{CF}), \left([0.4,0.7]e^{[0.5,0.7]\text{л}i}\right), \left(0.3e^{(0.6)\text{л}i}\right)\right), \\
\left((\text{MMI}, \text{MC}), \left([0.3,0.6]e^{[0.3,0.7]\text{л}i}\right), \left(0.3e^{(0.6)\text{л}i}\right)\right), \left((\text{MMI}, \text{SV}), \left([0.4,0.7]e^{[0.5,0.7]\text{л}i}\right), \left(0.3e^{(0.6)\text{л}i}\right)\right), \\
\left((\text{ABP}, \text{VW}), \left([0.5,0.8]e^{[0.3,0.4]\text{л}i}\right), \left(0.5e^{(0.7)\text{л}i}\right)\right), \left((\text{ABP}, \text{RW}), \left([0.5,0.7]e^{[0.4,0.5]\text{л}i}\right), \left(0.5e^{(0.7)\text{л}i}\right)\right), \\
\left((\text{ABP}, \text{CFI}), \left([0.3,0.7]e^{[0.4,0.7]\text{л}i}\right), \left(0.4e^{(0.7)\text{л}i}\right)\right), \left((\text{ABP}, \text{CF}), \left([0.4,0.8]e^{[0.4,0.7]\text{л}i}\right), \left(0.3e^{(0.6)\text{л}i}\right)\right), \\
\left((\text{ABP}, \text{BAC}), \left([0.3,0.7]e^{[0.4,0.7]\text{л}i}\right), \left(0.4e^{(0.7)\text{л}i}\right)\right), \left((\text{ABP}, \text{CSS}), \left([0.4,0.8]e^{[0.4,0.7]\text{л}i}\right), \left(0.3e^{(0.6)\text{л}i}\right)\right), \\
\left((\text{TE}, \text{VW}), \left([0.3,0.6]e^{[0.3,0.7]\text{л}i}\right), \left(0.3e^{(0.6)\text{л}i}\right)\right), \left((\text{TE}, \text{RW}), \left([0.4,0.7]e^{[0.5,0.7]\text{л}i}\right), \left(0.3e^{(0.6)\text{л}i}\right)\right), \\
\left((\text{TE}, \text{CFI}), \left([0.3,0.6]e^{[0.3,0.7]\text{л}i}\right), \left(0.3e^{(0.6)\text{л}i}\right)\right), \left((\text{TE}, \text{CF}), \left([0.4,0.7]e^{[0.5,0.7]\text{л}i}\right), \left(0.3e^{(0.6)\text{л}i}\right)\right), \\
\left((\text{TE}, \text{BAC}), \left([0.3,0.6]e^{[0.3,0.7]\text{л}i}\right), \left(0.3e^{(0.6)\text{л}i}\right)\right), \left((\text{TE}, \text{CSS}), \left([0.4,0.7]e^{[0.5,0.7]\text{л}i}\right), \left(0.3e^{(0.6)\text{л}i}\right)\right), \\
\left((\text{MC}, \text{VW}), \left([0.3,0.7]e^{[0.4,0.7]\text{л}i}\right), \left(0.4e^{(0.7)\text{л}i}\right)\right), \left((\text{MC}, \text{RW}), \left([0.4,0.8]e^{[0.4,0.7]\text{л}i}\right), \left(0.3e^{(0.6)\text{л}i}\right)\right), \\
\left((\text{MC}, \text{CFI}), \left([0.3,0.7]e^{[0.4,0.7]\text{л}i}\right), \left(0.4e^{(0.7)\text{л}i}\right)\right), \left((\text{MC}, \text{CF}), \left([0.4,0.8]e^{[0.4,0.7]\text{л}i}\right), \left(0.3e^{(0.6)\text{л}i}\right)\right), \\
\left((\text{MC}, \text{BAC}), \left([0.3,0.7]e^{[0.4,0.7]\text{л}i}\right), \left(0.4e^{(0.7)\text{л}i}\right)\right), \left((\text{MC}, \text{CSS}), \left([0.4,0.8]e^{[0.4,0.7]\text{л}i}\right), \left(0.3e^{(0.6)\text{л}i}\right)\right), \\
\left((\text{SV}, \text{VW}), \left([0.3,0.7]e^{[0.4,0.7]\text{л}i}\right), \left(0.4e^{(0.7)\text{л}i}\right)\right), \left((\text{SV}, \text{RW}), \left([0.4,0.8]e^{[0.4,0.7]\text{л}i}\right), \left(0.3e^{(0.6)\text{л}i}\right)\right), \\
\left((\text{SV}, \text{CFI}), \left([0.4,0.8]e^{[0.4,0.7]\text{л}i}\right), \left(0.3e^{(0.6)\text{л}i}\right)\right), \left((\text{SV}, \text{CF}), \left([0.3,0.7]e^{[0.4,0.7]\text{л}i}\right), \left(0.4e^{(0.7)\text{л}i}\right)\right), \\
\left((\text{SV}, \text{BAC}), \left([0.3,0.7]e^{[0.4,0.7]\text{л}i}\right), \left(0.4e^{(0.7)\text{л}i}\right)\right), \left((\text{SV}, \text{CSS}), \left([0.4,0.8]e^{[0.4,0.7]\text{л}i}\right), \left(0.3e^{(0.6)\text{л}i}\right)\right)
\end{array}\right\}$$

Just the effectiveness of security against a specific threat is shown by CCuFR. The first element in an ordering pair's M grade effect on the second element. Because of their constraints, these structures can only offer a limited amount of information. Meanwhile, the CCuqROFR gives the complete information.

### *5.2 Comparison on the Basis of Structure*

In this section, a comparison of the suggested methodologies with current methods is carried out. The CCuqROFSs and CCuqROFRs outperform all existing fuzziness-related theories and approaches. These sets explicitly cover M and non-M as two of the three groups. On the other hand, FRs, CuFRs, CCuFSs, IFRs, CuIFRs, CCuIFRs, PyFRs, CuPyFRs, CCuPyFRs, qROFRs, and CuqROFRs fail. All Comparison with CCuqROFSs dealing with the applications given in the below Table 4.

**Table 4**
Shows the comparative analysis on the basis of structure

| Structure | Membership | Non-Membership | Multidimensional | Dual Degree | Space |
|---|---|---|---|---|---|
| FR | Yes | No | No | No | $n = 1$ |
| CuFR | Yes | No | No | Yes | $n = 1$ |
| CCuFR | Yes | No | Yes | Yes | $n = 1$ |
| IFR | Yes | Yes | No | No | $n = 1$ |
| CuIFR | Yes | Yes | No | Yes | $n = 1$ |
| CCuIFR | Yes | Yes | Yes | Yes | $n = 1$ |
| PyFR | Yes | Yes | No | No | $n = 2$ |
| CuPyFR | Yes | Yes | No | Yes | $n = 2$ |
| CCuPyFR | Yes | Yes | Yes | Yes | $n = 2$ |
| qROFR | Yes | Yes | No | No | $n \geq 1$ |
| CuqROFR | Yes | Yes | Yes | No | $n \geq 1$ |
| CCuqROFR | Yes | Yes | Yes | Yes | $n \geq 1$ |

The proposed applications' supportive and discouraging effects on one another were discussed; these effects were represented by M and NM ratings, respectively. Also, although CCuPyFSs and CCuPyFRs can state both grades, they have serious problems. Table 4 makes it very evident that CuPyFSs fall short of solving the issue because the sum of the grades exceeds their tolerances. Although CuqROFSs have a greater range than CuPyFSs, they barely passed only four times in Table 4. Since the sum of the squares of the real and imaginary portions did not fall within the unit interval, CuqROFSs were also unable to solve the problem. As a result, we applied the notion to a wider range of applications in light of CCuqROFSs' dominance. Experts can freely display their discernment with CCuqROFSs. The CCuqROFRs structure is preferable, as seen above. It satisfies all five requirements, whereas the structures of its rivals are constrained.

## 6. Conclusion

In this paper, we explored the novel concepts of the complex cubic q-rung orthopair fuzzy set (CCuqROFS), complex cubic q-rung orthopair fuzzy relations (CCuqROFR), and Cartesian product of two CCuqROFSs. Moreover, numerous types of CCuqROFRs are also discussed, including CCuqRO-reflexive-FR, CCuqRO-irreflexive-FR, CCuqRO-symmetric-FR, CCuqRO-transitive-FR, CCuqRO-equivalence classes-FR, and many more. The CCuqROFS is the generalization form of the CuqROFS and IVCuqROFS. The more generalized version of each of the specified structures is the innovative idea behind the CCuqROFR. This structure defines all levels of both present and future aspects since it includes all levels, including M and NM with complicated numbers. They are more adept at handling fuzziness. These innovative frameworks and creative modelling techniques aim to address issues with web services security. The relationship between various risks and security measures is examined in the suggested study. They specify the levels of present and future effectiveness and ineffectiveness. These structures have the advantage of being utilised to define M and NM for all three stages with both the amplitude and phase terms. The various previous approaches are contrasted with the provided framework. The CCuqROFR is therefore more complex than all the other current structures. The CCuqROFR will be applied in the future for improved results. There are many intriguing and unique applications for this concept.

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**Conflicts of Interest**
The authors declare no conflicts of interest.

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