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Modeling Civilian and Militant Casualties in Asymmetric Wars: The Case of Gaza 2024

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ABSTRACT

Lanchester-type models for attritional warfare balance military casualties in two opposing forces. In asymmetric wars such as that in Gaza, by contrast, the dominant military force takes few casualties, and the crucial relationship is between casualties among opposing militants and among innocent civilians. We construct and analyze a simple dynamical model in which the proportion $\beta \in [0, 1]$ of effectively targeted, as opposed to indiscriminate, military actions determines the balance between militant and civilian casualties. We derive a conserved quantity which yields an analogue of Lanchester's laws for this balance, find the general solution of the model, and quantify the effects of variations in levels of targeting effectiveness on civilian casualties. Important conclusions are that every increase in β results in an approximately $\frac{1}{\beta(1-\beta)}$ times greater proportionate reduction in civilian casualties, and that, when militants are a small fraction of the population, the overall percentage of civilian casualties when the militant force has been eliminated is $\frac{1-\beta}{\beta}$ times the original percentage of militants in the population. We draw some insights regarding the 2023-2025 war in Gaza.

1. Introduction

Although data is scattered and considered unreliable [1] the estimated number of Palestinians killed by 10th December 2024 in the Gaza War, according to Palestinian sources, exceeds 44,000 [2]. While the stated targets are Hamas militants, many casualties are uninvolved civilians. In this Short Note we create a simple mathematical model for the dynamics of such conflicts and quantify the main factor that leads to this dire consequence.

Mathematical modelling of the dynamics of conflict goes back over a century [3,4]. One model-

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ing approach focuses on the mutual attrition of military forces, originating with [5]. Such models, in the form of differential equations, capture the instantaneous battle conditions as a result of attrition. Analysis seeks a conserved quantity, which gives insights into the integrated longer-term outcomes of the conflict. While Lanchester's original models consider two homogeneous regular forces in symmetrical combat situations, Deitchman [6] extended them to guerrilla warfare where the battle conditions are asymmetrical, in that while the combatants of the regular force are exposed, the guerrillas benefit from cover and concealment. A body of 'Lanchester theory' extends this military perspective to a range of other force structures and battle conditions [7].

Another modeling approach for conflicts originates with the physicist and pacifist Lewis Fry Richardson. In a long series of Letters to *Nature* [8] culminating in two books [9,10] he shifted the perspective to the phenomenon of war, its prevention, and its effects on innocent civilians, effectively founding the quantitative study of peace. From this perspective, the Deitchman model neglects the central fact that it is the targeting inaccuracy or laxity of the regular force that results in casualties among the civilian population in which the guerrillas are diffused.

The wars in Iraq and Afghanistan are recent examples of such situations, but the war in Gaza is a much more extreme case. Further, while the Israeli Defense Force (IDF) has suffered some attrition too (a few hundred casualties) this has been relatively small and has not affected IDF capabilities. Thus, while it is perfectly possible to apply Deitchman's model or other Lanchestrian ideas to the Gaza war [11,12], this would not help us in our purpose, which is to analyze the relationship between militant and civilian casualties. A detailed attempt to do so for the longer-term, sporadic 2008–2023 Gaza conflict looks at combatant and civilian hazards of death and their ratio, but within a homogeneous population [13]. The same longer-term conflict is analyzed game-theoretically in [14], and found in [15] to fit into the same pattern of power-law-distributed events as other inusurgencies.

We deploy Lanchester-like theory in the spirit of Richardson's analyses, to understand the effects of an attritional asymmetric war on a civilian population [16]. We write down a simple dynamical system which embodies the essential reason for the deaths of non-combatants, solve the model, construct its conserved quantity, and thereby identify the relationship between targeting accuracy and the balance of civilian and militant casualties. Although we offer some insights about the Gaza war based on limited publicly available data sources, our main purpose is to provide a simple quantitative model of such a conflict's core dynamics. The aim is to focus attention on the problem and its modeling possibilities and thereby stimulate wider analysis, not necessarily to capture the full complexity of the situation, nor to make any specific predictions or policy recommendations.

2. A Dynamical Model for Militant and Civilian Casualties

Let M and C denote the guerrilla Militants and the Civilians respectively within a total population $N = M + C$. In the context of Gaza, M represents the Hamas militants and C the Palestinian non-combatant civilians. The attritional dynamics of the combat situation are governed by two parameters: the fire intensity α of the opposing regular ('State') force (in Gaza, the IDF), and its targeting accuracy $\beta \in [0, 1]$, which is the fraction of fire α that is accurately aimed at M. When $\beta = 1$, the State's fire is perfectly accurate and civilians are unharmed. If $\beta = 0$ then no targeting information is available and the State's fire is entirely random. In that case, the number of militant casualties is proportional to their fraction in the population, and civilians are hit too – in great numbers when the militant fraction is small. This attrition is represented by the differential equations

$$\frac{dM}{dt} = -\alpha\left(\beta + (1-\beta)\frac{M}{N}\right), \tag{1}$$

$$\frac{dC}{dt} = -\alpha\left((1-\beta)\frac{C}{N}\right). \tag{2}$$

Note that the combat aspects of the State forces – which include their numbers, per capita effectiveness, intensity of action, and so on – are embedded in the parameter α. Absent significant attrition to the State forces, α may vary over time for exogenous reasons, which may be tactical or political, rather than due solely to force availability. The crucial aspects of intelligence, situational awareness, rules of engagement and (as a result) targeting accuracy are embedded in the parameter β.

3. Results

3.1 *Casualty Exchange Ratio*

With $0 < \beta < 1$, dividing (1) by (2) we obtain the instantaneous ratio of militant to civilian casualties,

$$\frac{dM}{dC} = \frac{M + \beta C}{(1-\beta)C}. \tag{3}$$

Notice the cancellation of the State's combat intensity factor α, on which this ratio no longer depends.

3.2 *Lanchester-type conservation law*

Unlike in Lanchester's model this differential equation is not separable, but in the form

$$\frac{dM}{dC} - \frac{1}{1-\beta}\frac{M}{C} = \frac{\beta}{1-\beta} \tag{4}$$

it can be integrated by multiplying by $C^{-\frac{1}{1-\beta}}$, so that it becomes

$$\frac{d}{dC}\left(C^{-\frac{1}{1-\beta}}M\right) = \frac{\beta}{1-\beta}C^{-\frac{1}{1-\beta}}. \tag{5}$$

Integrating this from initial values $M = M_0, C = C_0$ gives

$$C^{-\frac{1}{1-\beta}}M - C_0^{-\frac{1}{1-\beta}}M_0 = -C^{1-\frac{1}{1-\beta}} + C_0^{1-\frac{1}{1-\beta}} \tag{6}$$

and thus the Lanchester-type conservation law

$$C^{-\frac{1}{1-\beta}}(M + C) = C_0^{-\frac{1}{1-\beta}}(M_0 + C_0) \tag{7}$$

or more simply, in terms of C and $N = M + C$,

$$\frac{C^{\frac{1}{1-\beta}}}{N} = \frac{C_0^{\frac{1}{1-\beta}}}{N_0} \tag{8}$$

where $N_0 = M_0 + C_0$ is the total initial population.

Lanchester laws are often used to give a criterion for complete elimination of one's enemy. Although such an outcome is typically unrealistic, we note that we can re-write (8) as

$$M = \left(\frac{C}{C_0}\right)^{\frac{1}{1-\beta}} N_0 - C, \tag{9}$$

and we observe that militant numbers are zero when (writing the final value of C at this time as C_F)

$$\left(\frac{C_F}{C_0}\right)^{\frac{\beta}{1-\beta}} = 1 - \frac{M_0}{N_0}. \tag{10}$$

This has an especially simple interpretation when the militants are a small proportion of the population, $M_0 \ll N_0$, for then the overall percentage of civilian casualties when the militant force has been destroyed is $\frac{1-\beta}{\beta}$ times the original percentage of militants in the population.

To give a concrete example, in Fig.1 we show a phase portrait of the system for $\beta = \frac{1}{3}$ with a trajectory plotted from $C_0 = 400, M_0 = 400$ — that is, one third of state action is accurately targeted at militants, while the remainder is against the full population of whom the militants initially form half. The trajectory flattens out as numbers decline, with significantly more militant casualties per civilian casualty as the end state (10), $C = 100, M = 0$ is approached.

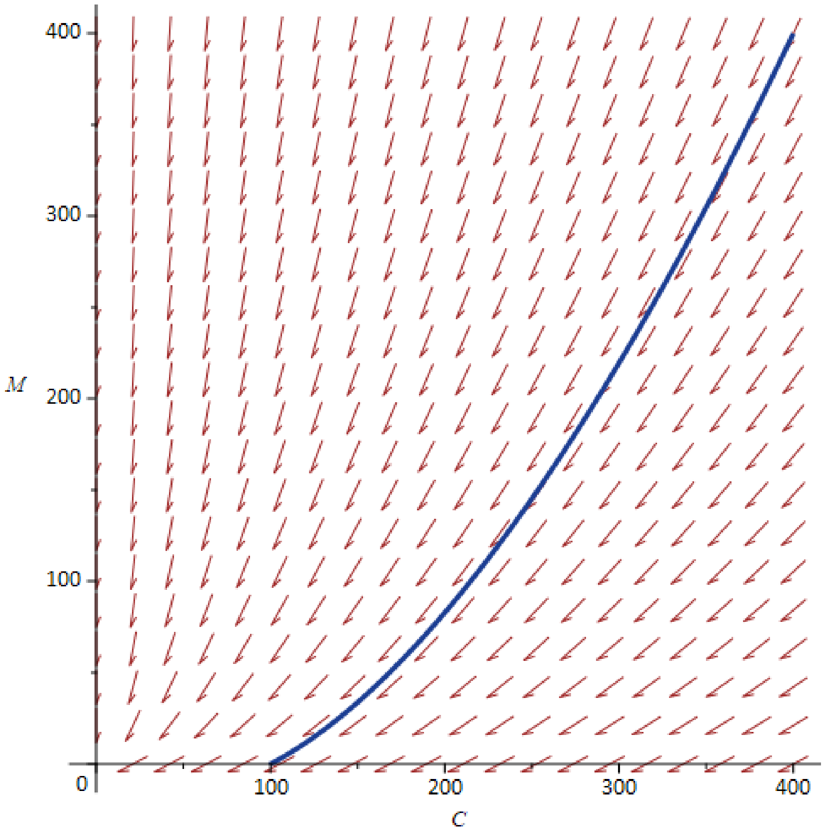

Fig. 1. Phase portrait of Civilian C and Militant M numbers under the model with $\beta = \frac{1}{3}$ (and $\alpha = 10$, which does not affect the plot). Trajectory is from $C_0 = 400, M_0 = 400$ and terminates at $C = 100, M = 0$

The total population of Gaza at the beginning of the conflict was estimated as $N_0 = 2.1$ million [17], among them M_0 = 30,000–40,000 Hamas militants [18], so that $M_0/N_0 \simeq 0.02$ at most. According to

Palestinian sources the total number of fatalities in mid-December 2024 was estimated at 44,800 [2]. The number of Hamas fatalities is impossible to estimate well, since Hamas does not provide figures for its military fatalities and denies Israeli estimates, which in August 2024 reached 17,000 [19]. Based on these highly uncertain estimates, and to exemplify our results, we take $\beta \simeq 1/3$. It is entirely possible that the true β may be lower [20].

3.3 *Sensitivity to variation in* β

The fatality ratio (3) is highly sensitive to changes in β, with the partial derivative

$$\frac{\partial}{\partial \beta}\left(\frac{dC}{dM}\right) = -\frac{NC}{(\beta C + M)^2}. \tag{11}$$

When, as is the case in Gaza, M is a small proportion of C (written $M \ll C$), this is approximately $-\frac{1}{\beta^2}$, and (taking $\frac{1}{\beta} = 3$) the ratio of rates of Civilian to Militant casualties is reduced by approximately 9% for every 1% replacement of untargeted fires by targeted. The inverse-β^2 scaling is especially sensitive at small β, and if the true β is lower then this ratio will be much higher.

However, when M/N is not small, the full dynamics of the model applies. Arguably, the actual number C^* of civilians affected is almost never the full civilian population C, since any individual IDF action is geographically located and therefore there are $C - C^*$ civilians with probability zero to be hit. For example, if $M/N = 0.2$ then the 9% reduction in civilian casualties noted above is reduced to 3.7%.

The surviving final civilian population C_F when $M = 0$ (10) is similarly sensitive to β, with

$$\frac{\partial}{\partial \beta}\left(\frac{C_F}{C_0}\right) = -\frac{1}{\beta(1-\beta)}\frac{C_F}{C_0}\log\frac{C_F}{C_0}. \tag{12}$$

It follows that every increase in β results in an approximately $\frac{1}{\beta(1-\beta)}$ times greater proportionate reduction in civilian casualties. Thus, in Gaza, based on the available data, every switch of 1% of IDF fires from untargeted to targeted on militant Hamas would reduce the total civilian casualties by roughly 4.5%. If effective C^* were significantly smaller than nominal C, then this ratio would be lower. On the other hand, if β were significantly lower than estimated then it could be much higher. Such uncertainties should not, however, distract from the importance of the effect.

3.4 *General solution of the model*

Although our deductions are made from the casualty ratio (3) and its integrated conservation law (8) and state equation (10), for completeness we provide solutions of the model. To do so we first note that $N(t)$ decays linearly: the sum of (1) and (2)

$$\frac{dN}{dt} = \frac{dM}{dt} + \frac{dC}{dt} = -\alpha \tag{13}$$

so that $N(t) = N_0 - \alpha t$. Then (1) becomes

$$\frac{dM}{dt} + \frac{(1-\beta)\alpha}{N_0 - \alpha t}M = -\beta\alpha, \tag{14}$$

or equivalently

$$\frac{d}{dt}\left((N_0 - \alpha t)^{\beta-1}M\right) = -\beta\alpha(N_0 - \alpha t)^{\beta-1}, \tag{15}$$

so that

$$(N_0 - \alpha t)^{\beta-1} M - N_0^{\beta-1} M_0 = -\beta\alpha \int_0^t (N_0 - \alpha\tau)^{\beta-1}\, d\tau = (N_0 - \alpha T)^{\beta} - N_0^{\beta} \tag{16}$$

or

$$M = N_0 - \alpha t - \left(1 - \frac{\alpha t}{N_0}\right)^{1-\beta} (N_0 - M_0). \tag{17}$$

In a similar manner from (2), or simply as $C = N - M$, we derive

$$C = \left(1 - \frac{\alpha t}{N_0}\right)^{1-\beta} C_0. \tag{18}$$

It can easily be verified that (17,18) satisfy (3) and (7), and that they satisfy (10) at time T, where

$$T = \frac{N_0}{\alpha} \left\{ 1 - \left(1 - \frac{M_0}{N_0}\right)^{\frac{1}{\beta}} \right\}. \tag{19}$$

For $M_0 \ll N_0$ this simplifies to

$$T \simeq \frac{M_0}{\alpha\beta}. \tag{20}$$

These results generalize to time-varying $\alpha(t)$. Suppose that α takes the initial value $\alpha(0) = \alpha_0$. It is then straightforward to adapt the solution above into a solution for $\alpha(t) = \alpha_0 f(t)$ for any smooth function f, since the solution $y(t)$ of

$$\frac{dy}{dt} = \alpha_0 g(y) \tag{21}$$

for any smooth function g immediately furnishes a solution $y\left(F(t)\right)$ of the differential equation

$$\frac{dy}{dt} = \alpha(t) g(y) \equiv \alpha_0 f(t) g(y) \tag{22}$$

where

$$F(t) = \int_0^t f(\tau)\, d\tau. \tag{23}$$

The solutions (17,18) then become

$$M = N_0 - \alpha_0 F(t) - \left(1 - \frac{\alpha_0 F(t)}{N_0}\right)^{1-\beta} (N_0 - M_0) \tag{24}$$

$$C = \left(1 - \frac{\alpha_0 F(t)}{N_0}\right)^{1-\beta} C_0. \tag{25}$$

As an example, suppose α decays exponentially at rate r, so that $\alpha(t) = \alpha_0 e^{-rt}$, and thence $F(t) = (1 - e^{-rt})/r$. If $r < \alpha_0\beta/M_0$ then $M = 0$ at

$$F(T) = (1 - e^{-rT})/r = \frac{N_0}{\alpha_0} \left(1 - \left(1 - \frac{M_0}{N_0}\right)^{\frac{1}{\beta}}\right) \tag{26}$$

or

$$T = -\frac{1}{r} \log\left(1 - \frac{rN_0}{\alpha_0}\left(1 - \left(1 - \frac{M_0}{N_0}\right)^{\frac{1}{\beta}}\right)\right), \tag{27}$$

which is finite provided

$$1 - \frac{\alpha_0}{rN_0} < \left(1 - \frac{M_0}{N_0}\right)^{\frac{1}{\beta}}, \tag{28}$$

that is to say if r is sufficiently small.

For $M_0 \ll N_0$, this threshold is $r = \alpha_0\beta/M_0$. If $r < \alpha_0\beta/M_0$, the time is then given by (20). If $r > \alpha_0\beta/M_0$ but $\frac{\alpha_0}{rN_0} \ll 1$ then

$$\lim_{t\to\infty} M = N_0 - \frac{\alpha_0}{r} - \left(1 - \frac{\alpha_0}{rN_0}\right)^{1-\beta}(N_0 - M_0) \simeq M_0 - \frac{\alpha_0\beta}{r}. \tag{29}$$

4. Conclusions

Lanchester theory, and its associated 'laws', are useful tools for thinking about how the conditions of warfare affect its outcomes. To our knowledge, such a step has never been made for civilian casualties and their relation to militant casualties when an overwhelming state force attempts to eliminate a militant sub-population. Our principal contribution has been to provide a simple model (1,2) of this relationship and its central cause, the distinction between fractions of discriminating (β) and indiscriminate ($1-\beta$) military action. We wrote down an analogue of the Lanchester and Deitchman laws which captures the development of the relationship over time (8), together with solutions of the equations (24,25) and an analysis of sensitivity to variation in the accuracy of targeting. The most important outcomes were that every increase in β results in an approximately $\frac{1}{\beta(1-\beta)}$ times greater proportionate reduction in civilian casualties, and that the overall percentage civilian casualties when a small militant fraction has been eliminated is $\frac{1-\beta}{\beta}$ times the original percentage of militants in the population.

In the Gaza war a comparatively small number of Hamas militants are embedded in a much larger civilian population, and β is likely small. If $\beta \simeq \frac{1}{3}$ then, in our model, final percentage civilian casualties would be double the initial militant percentage of the population. Further, if around 30% of current Israeli military actions are accurately targeted then every additional 1% would lead to up to nine fewer civilian casualties for every militant casualty, and to an overall civilian death toll about 4.5% lower. If Israel's claims are inflated, and true targeting accuracy is lower, these ratios could be significantly higher.

Finally, the model provides a basis for incorporation of many further effects. Most obviously, heterogeneity of forces, populations and geography would be essential features of any predictive model. The model could be tensioned against militant decisions in a game-theoretic manner, with β becoming an endogenous decision variable [21]. Trade-offs between α and β for the state forces' utility could be considered [22]. However, the core model's simplicity has as its foremost merit that it enables discussion of a connection which previously remained unanalyzed [23; 10, p169].

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Conflicts of Interest

The authors declare no conflicts of interest.

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