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Geometric Approach for Solving First Order Non-Homogenous Fuzzy Difference Equation

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ABSTRACT

Real-life scenario modeling with mathematics is very important nowadays. Depending upon system behavior, it may also model discrete systems in several cases. In discrete system modeling, the difference equation is one of the well-known methodologies. However, if some uncertain factors are involved in the discrete models, then uncertain difference equation concepts come into play. The fuzzy difference equation is one of them. The fuzzy difference equation is significant, as it can represent variances of dependent variables in a discrete frame under uncertainty. In this paper, a first-order non-homogeneous linear difference equation is considered under fuzzy uncertainty, a special kind of fuzzy difference equation. Here, a well-known fuzzy geometric approach is utilized to solve the mentioned first-order non-homogeneous fuzzy difference equation. An application, namely a fuzzy prescription for digoxin based on the fuzzy initial value problem, is also discussed in numerical illustrations as a consequence of the proposed theory..

1. Introduction

Classical set theory has the limitation that the set corresponds to a characteristic function with the values 0 and 1 representing belongingness and non-belongingness. However, it is evident from many real-life scenarios that only these two extreme values cannot explain many grey shades between extremities. In this context, Zadeh [1] introduced a fuzzy set to clarify the situation with a practical and acceptable idea compared to the classical set. The membership function lies in the interval [0, 1] in this set theory concept. The intermediate decisions of certain and certain situations take some membership value. Furthermore, several researchers [2-7] have investigated fuzzy set theory, fuzzy numbers, and their application in different fields of mathematics.

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1.1 Difference equation

Qualitative analysis of discrete phenomena using different equations becomes contemporary. A difference equation is an equation that specifies the change of the variables between two periods. The discrete dynamical phenomena are best described mathematically using the difference equation. The difference equation, or a system of difference equations used to represent a particular dynamical situation, comprises several coefficients, parameters, and beginning conditions.

1.2 Fuzzy difference and differential equations

The solutions provided by the clean environment frequently include errors, which makes them describe reality more accurately than they should. We encounter these situations by incorporating fuzzy set theory into different equations. A difference equation can be regarded as a fuzzy difference equation in any of the below-mentioned characteristics as fuzzy numbers:

i. Only the initial information,
ii. Only the associated coefficients,
iii. Both information and the coefficients.

Deeba *et al.,* [8] investigated the global behavior of the first-order linear fuzzy difference equation whereas Khastan [9] studied the existence, uniqueness, and global behavior of the solutions of the different formulations of a fuzzy difference equation. Alamin *et al.,* [10,11] solve analytically with stability analysis and its applications of the fuzzy linear in-homogeneous and homogeneous models in neutrosophic environments, respectively. Mondal *et al.,* [12] have discussed the existence and stability of the solution of the first-order homogeneous linear difference equation. Rahaman *et al.,* [13,14] solved linear difference equations by interval method and Gaussian fuzzy number. Many researchers worked on theoretical and applications-based fuzzy equations, fuzzy difference equations, and fuzzy differential equations in their articles (see [15-26]).

1.3 Motivations and novelty

We have seen in the short literature history that researchers studied the fuzzy difference equation in various ways, techniques, and perspectives. Researchers always try to find new ways or possibilities to solve a problem and understand the merits or demerits of the discussed method by comparing it with the previous one. The fuzzy geometric method is very useful to solve the fuzzy differential equation problem in several cases. However, no one still uses the easy and useful straightforward method to solve fuzzy difference equations. We have solved the linear first order in-homogeneous fuzzy difference equation through a geometric approach and its effect on the solution dynamics. We have solved this fuzzy initial valued difference equation by dividing it into two parts. One is an inhomogeneous crisp problem, and another is a homogeneous, purely fuzzy initial value problem. Also, we applied practical life problems to show the strategy's applicability with numerical illustrations.

2. Preliminaries

2.1 Fuzzy set theory

The fuzzy set explains the ambiguous situation of belongingness through the gradation value within the set $[0,1]$, which may contain any one point of this interval. Whenever, in a crisp sense, the membership value takes either 0 or 1. But logically and practically, the fuzzy set concept is more acceptable than the classical set theory. Professor L. Zadeh gave this innovative concept [1]. We try to clear the facts through an example stated below:

Example 1: Suppose a variety of color roses in a garden is considered a universal set, with pink color roses as a subset, and in our perception, we choose a very deep pink colored rose as perfect.

Among the pink roses, there may be such types of variation as very deep pink, deep pink, near about deep pink, pink, slightly pink, and different from pink. Thus, the gradation values of the roses in the pink color roses set take any value within $[0,1]$ according to the perception of the observer. Table 1 describes the phenomena.

Table 1
Comparison of gradation values between fuzzy set and crisp set

	Types of color of roses	Gradation values	
		Fuzzy	Crisp
1.	Very deep pink	1	1
2.	Deep pink	0.8	0
3.	Near about deep pink	0.7	0
4.	Pink	0.6	0
5.	Slightly pink	0.4	0
6.	Totally different from pink	0	0

Now, some definitions and conceptual results on fuzzy set theory and fuzzy numbers are given below:

Fuzzy set: [1] In the universe of discourse X, a fuzzy set $\tilde{A}$ can be defined by the order pair $(x, \mu_{\tilde{A}}(x))$, the first component is the element in X, and the second is the corresponding membership grade.

α*- level set:* [27] The collection of all elements of a given fuzzy set that has a membership grade greater than α is called a α-cut of the fuzzy set $\tilde{A}$ and is denoted by A_α.

Fuzzy number (Triangular): [28] A 3- tuple number, $\widetilde{Q} = (q_1\ q_2, q_3)$ is a triangular fuzzy number, and its corresponding membership function is defined by

$$\mu_{\widetilde{Q}}(x) = \begin{cases} 0 & for\ x < q_1 \\ \dfrac{x - q_1}{q_2 - q_1} & for\ q_1 \leq x < q_2 \\ 1 & for\ x = q_2 \\ \dfrac{q_3 - x}{q_3 - q_2} & for\ q_2 < x \leq q_3 \\ 0 & for\ x > q_3 \end{cases}$$

In geometric approach, the trapezoidal fuzzy number $\widetilde{Q} = (q_1\ q_2, q_3)$ can be expressed as $\widetilde{Q} = Q_{cp} + \widetilde{Q}_{up}$ (certainty part + uncertainty part). The certainty part Q_{cp} is the crisp value q_2 where the membership value is always one. The uncertainty fuzzy number $\widetilde{Q}_{up} = (q_1 - q_2, 0,\ q_3 - q_2)$ is a triangular fuzzy number.

Hukuhara-difference [29]: A fuzzy number $\widetilde{w}$ is said to be a Hukuhara difference between two fuzzy numbers $\tilde{s}$ and $\tilde{t}$ if it satisfies the fuzzy equation $\tilde{s} = \widetilde{w} + \tilde{t}$.

3. Comparative Study on Geometric Approach for Fuzzy Problem

The geometric approach for solving fuzzy-based problems is not new. The comparative studies between related works are given in Table 2.

Table 2
Comparison among the existing works and contribution of this article.

Sl. No.	Paper details	Main topic	Discrete or Continuous system	Application or theoretical contribution
1.	Gasilov *et al.,* [30]	Solution of linear differential equations with fuzzy boundary values	Continuous	Theoretical
2.	Gasilov *et al.,* [31]	Solution of boundary value problem with fuzzy forcing function	Continuous	Application and theoretical both
3.	Gasilov *et al.,* [32]	Solution of fuzzy differential equation with initial value	Continuous	Theoretical
4.	Gasilov *et al.,* [33]	Solution of linear differential equations with fuzzy boundary values	Continuous	Theoretical
5.	Gasilov *et al.,* [34]	Solution of fuzzy linear systems of differential equation	Continuous	Theoretical
6.	Gasilov *et al.,* [35]	Solving fuzzy linear systems of equation	Continuous	Theoretical
7.	This paper	Solving fuzzy difference equation (in-homogeneous first order linear)	Discrete	Application and theoretical both

4. Geometric approach for the fuzzy difference equation

The geometric method for solving fuzzy problems is not new [30-35]. Here, the approach is applied to solving fuzzy difference equations.

Let us consider an initial value problem consisting of a first-order linear in-homogeneous difference equation with a constant coefficient in which the initial value is a fuzzy number as

$$\begin{cases} u_{n+1} - au_n = g(n) \\ u_{n=0} = \widetilde{U}_0 \end{cases} \tag{1}$$

Here $g(n)$ is a crisp function of the iteration number n.

The initial value $\widetilde{U}_0$ is a triangular fuzzy number and can be expressed as $\widetilde{U}_0 = \mathcal{U}_0 + \widetilde{\mathcal{U}}_0$, where $\mathcal{U}_0$ and $\widetilde{\mathcal{U}}_0$ are crisp and triangular fuzzy numbers, respectively.

By the concept of geometric approach, we separate the problem (1) in the following way:

(a) Crisp in-homogeneous initial valued difference equation:

$$\begin{cases} u_{n+1} - au_n = g(n) \\ u_{n=0} = \mathcal{U}_0 \end{cases} \tag{2}$$

and

(b) Homogeneous fuzzy initial valued difference equation:

$$\begin{cases} u_{n+1} - au_n = 0 \\ u_{n=0} = \widetilde{\mathcal{U}}_0 \end{cases} \tag{3}$$

Therefore, the solution of equation (1) is $\tilde{u}_n = {}^{cp}_{n}u + {}^{fp}_{n}\tilde{u}$, where the crisp solutions ${}^{cp}_{n}u$ of equation (2) can be obtained easily, and the fuzzy solution of equation (3) is ${}^{fp}_{n}\tilde{u} = a^n\widetilde{\mathcal{U}}_0$. The coefficient of equation (3) is regarded as a positive crisp quantity.

Note 4.1:

(i) a^n is the only independent solution of (3). Therefore, the fuzzy solution ${}^{fp}_{n}\tilde{u} = \{a^n c_0 : c_0 \in \widetilde{\mathcal{U}}_0\}$ and its membership function $\mu_{{}^{fp}_{n}\tilde{u}}(a^n c_0) = \mu_{\widetilde{\mathcal{U}}_0}(c_0)$.

(ii) The initial value $\widetilde{u}_0$ is given either as a triangular fuzzy number or in a parametric representation of fuzzy number, the solution is always ${}^{fp}_{n}\tilde{u} = a^n \widetilde{u}_0$.

5. Numerical illustration

Now, we investigate the solution of a first-order linear in-homogeneous difference equation through the mentioned method.

Example 2: Solve the fuzzy initial valued difference equation

$$\begin{cases} u_{n+1} - 2u_n = n \\ u_{n=0} = \tilde{u}_0 = (2, 3, 4) \end{cases} \tag{4}$$

We express the initial value $\tilde{u}_0 = (2,\ 3,\ 4) = 3 + (-1, 0, 1)$ that is in terms of a certain and purely uncertain portion.

Now, we solve the crisp, in-homogeneous problem

$$\begin{cases} u_{n+1} - 2u_n = n \\ u_{n=0} = u_0 = 3 \end{cases} \tag{5}$$

The auxiliary equation of the corresponding homogeneous problem is $\lambda - 2 = 0$. Therefore, $\lambda = 2$. The complementary part of the solution is $k2^n$, k is an arbitrary constant to be found using the initial condition.

The particular integral of the problem (5) is

$\frac{1}{(E-2)}(n)$, E is the shift operator

$= -(1 - \Delta)^{-1}(n)$, as $E = 1 + \Delta$, Δ being the difference operator.

$= -(n + 1)$

So, ${}^{cp}_{n}u = k2^n - (n + 1)$ (6)

If we put the initial information as $u_0 = 3$, the obtained result should be

$${}^{cp}_{n}u = 4 \times 2^n - (n + 1) \tag{7}$$

The solution of the homogeneous fuzzy initial valued difference equation

$$\begin{cases} u_{n+1} - 2u_n = 0 \\ u_{n=0} = \widetilde{u}_0 = (-1, 0, 1) \end{cases} \tag{8}$$

As the imprecise part only ${}^{fp}_{n}\tilde{u} = 2^n(-1, 0, 1)$.

Finally, the solution of the problem is

$$\begin{aligned} \tilde{u}_n &= {}^{cp}_{n}u + {}^{fp}_{n}\tilde{u} = 4 \times 2^n - (n + 1) + 2^n(-1, 0, 1) \\ &= 2^n(3, 4, 5) - (n + 1) \end{aligned} \tag{10}$$

In α- cut representation of the equation (10), we have,

$$[u_n^L(\alpha), u_n^R(\alpha)] = 2^n[3 + \alpha, 5 - \alpha] - (n + 1) \tag{11}$$

The solution of the equation (7) and equation (11) are given in Figures 1, 2, 3, and 4, for $\alpha = 0, 0.5, 0.8$, and 1, respectively.

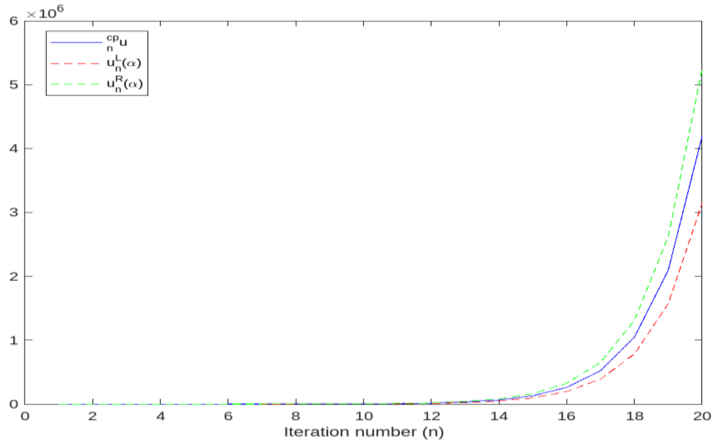

Fig.1. Solution of equation (7) and equation (11) for $\alpha = 0$.

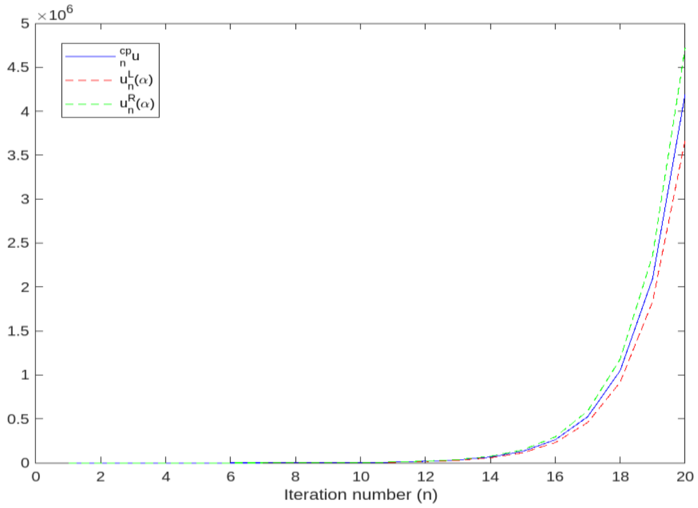

Fig. 2. Solution of equation (7) and equation (11) for $\alpha = 0.5$

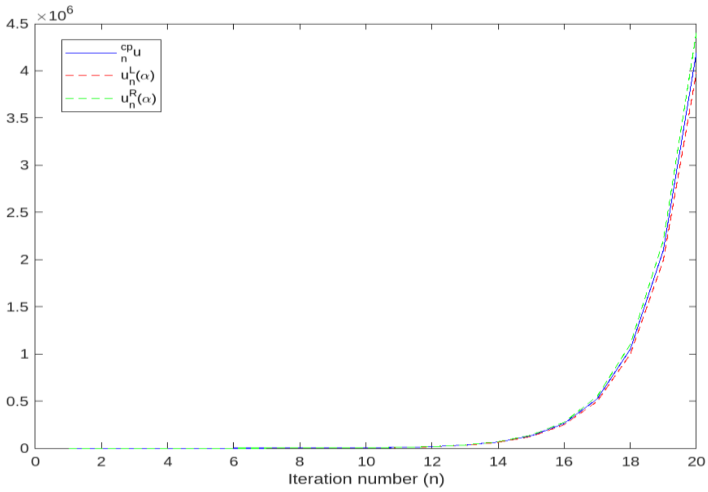

Fig. 3. Solution of equation (7) and equation (11) for $\alpha = 0.8$

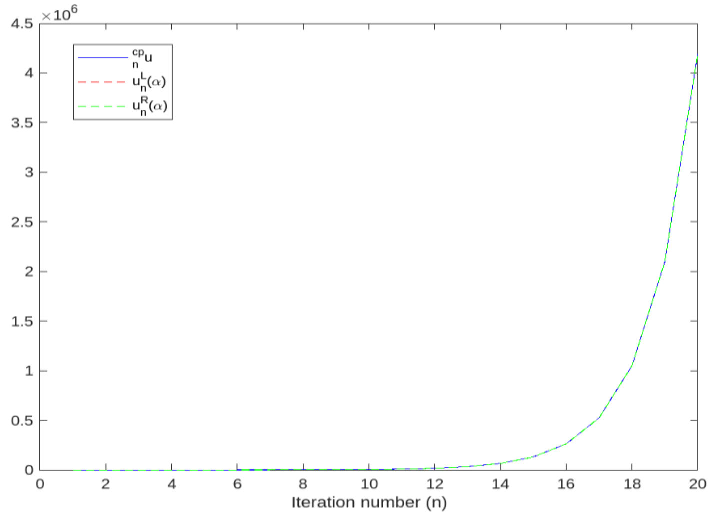

Fig. 4. Solution of equation (7) and equation (11) for $\alpha = 1$

In Figures 1-4, the blue line represents the crisp solution of the corresponding problem, and the dashed lines indicate α- cuts corresponding to the uncertain fuzzy solution. Both the fuzzy and crisp solutions are unbounded. If the uncertainty tends to zero, the corresponding solution approaches towards a crisp solution.

6. Application of the theory for prescription for digoxin

It is to be mentioned that digoxin is a medication that deals with cardiac problems. Here, the fuzzy difference equation considers the gradual depletion of the digoxin level in blood and maintaining its acceptable level for safe and effective livelihood. For mathematical model construction, we consider a hypothetical phenomenon as follows: Let us suppose given a prescription of daily drug dosage is about 0.1 and 0.5, part of the drug dosage remains in the system at the end of the period. Furthermore, initial concentration is supposed to be given by triangular fuzzy data like $D_{n=0} = (0.25,0.3,0.40)$. So, the governing mathematical model can be represented by following the fuzzy difference equation with initial information:

$$\begin{cases} \widetilde{D}_{n+1} = 0.5\widetilde{D}_n + 0.1 \\ \widetilde{D}_{n=0} = (0.25,0.3,0.40) \end{cases} \tag{12}$$

The initial value $\widetilde{D}_0 = 0.3 + (-0.05, 0, 0.1)$

Applying the proposed approach and following the similar computation procedure of the numerical example 5.1, we have the solution of equation (12) as

$$\widetilde{D}_n = {}^{cp}_{n}D + {}^{fp}_{n}\widetilde{D} = 0.1 \times 2^{-n} + 0.2 + 2^{-n}(-0.05, 0, 0.1)$$

$$\text{Or, } \widetilde{D}_n = 0.2 + 2^{-n}(0.05, 0.1, 0.2) \tag{13}$$

Therefore, the α- cut representation of the equation (13) is

$$[D_n^L(\alpha), D_n^R(\alpha)] = 0.2 + 2^{-n}[0.05 + 0.05\alpha, 0.2 - 0.1\alpha] \tag{14}$$

Where $\alpha \in [0,1]$. The solution of equation (14) is given in Figures 5 and 6, for $\alpha = 0$ and 0.85, respectively. Furthermore, the solution of equation (14) is given in Figures 7 and 8, for $n = 8$ and 12, respectively.

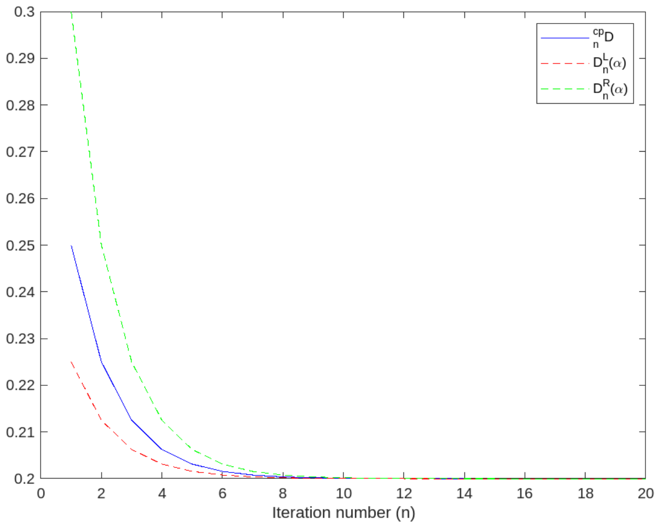

Fig. 5. Solution of equation ${}^{cp}_{n}D = 0.1 \times 2^{-n} + 0.2$ and equation (14) for $\alpha = 0$

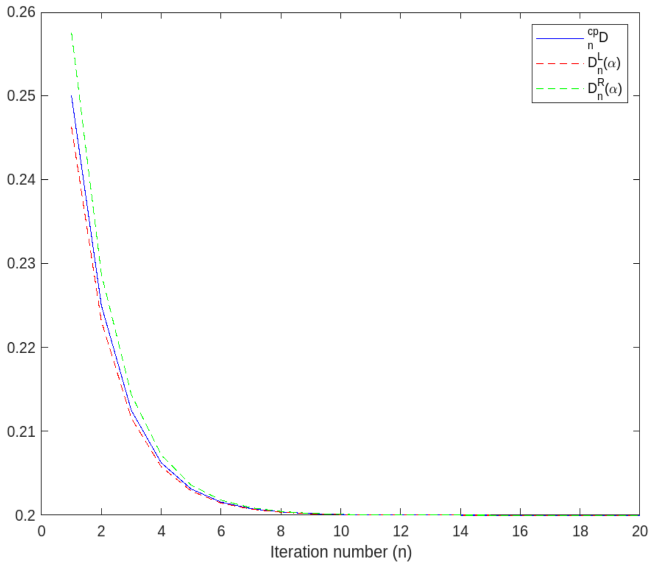

Fig. 6. Solution of equation ${}^{cp}_{n}D = 0.1 \times 2^{-n} + 0.2$ and equation (14) for $\alpha = 0.85$

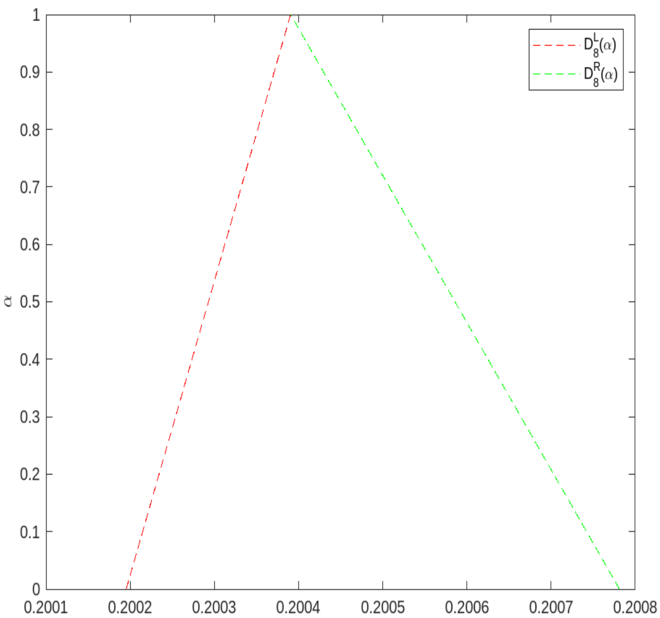

Fig. 7. Solution of equation (14) for $n = 8$

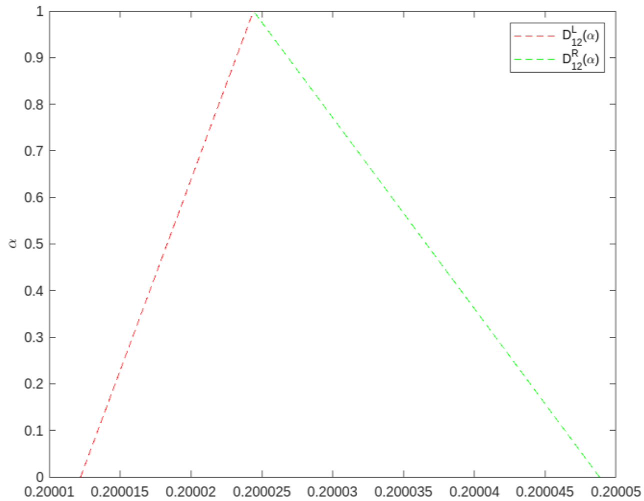

Fig. 8. Solution of equation (14) for $n = 12$

Figures 5 and 6 reflect those three types of solutions, one for crisp solution and another two branches are α-cut of fuzzy solution of equation (12) are convergent and converges to 0.2, which is the integral of the crisp inhomogeneous problem of (12). Figures 7 and 8 are the triangular fuzzy solution of equation (13) for $n = 8$ and $n = 12$, respectively. For larger n, the solution corresponding to $\alpha = 1$ tends to 0.2.

6. Conclusion and future research scope

Difference equations play a significant role in discrete dynamical systems. Uncertainty is also an integral part of many real physical phenomena. Fuzzy difference equations are crucial for modeling discrete systems where uncertainty, vagueness, or imprecision is dominant. It applies in fields like economics, engineering, biology, management and social sciences. By incorporating fuzzy sets concepts, these difference equations allow for more accurate illustration and estimate of solutions of a particular model in discrete systems. This framework gives a decision-making concept under uncertainty, provides sensitivity analysis, and extends the classical difference equation theories, contributing to practical applications and theoretical advancements for understanding complex problems. In this paper, we have solved a linear order non-homogenous difference equation under a fuzzy setting. We have utilized a geometric approach for solving the fuzzy difference equation, which may be regarded as a new approach in the theory of difference equations under fuzzy impreciseness. The proposed theory is used in the prescription of digoxin problems, and it is observed that the approaches may be applied as mathematical tools to deal with similar situations.

The theory manifested in this paper can be extended to the difference equation system under fuzzy-based uncertainty, which incurs more complexities. Further theoretical development may include studying stability, chaos, and bifurcation with high-dimensional or nonlinear systems in fuzzy difference equation systems. Furthermore, the discrete system can follow the proposed approach in imprecise environments, including intuitionistic fuzzy, neutrosophic, and type 2 interval environments. The future research scope for fuzzy difference equations as application is broad and very promising, with potential progressions in several application areas like population dynamics, climate modeling, network dynamics, and financial risk management problems.

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Conflicts of Interest

The authors declare no conflicts of interest.

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